Risk of Rare Disasters, Euler Equation Errors and the Performance of the C-CAPM

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Abstract

This paper shows that the consumption-based asset pricing model (C-CAPM) with low-probability disaster risk rationalizes large pricing errors, i.e., Euler equation errors. This result is remarkable, since Lettau and Ludvigson (2009) show that leading asset pricing models cannot explain sizeable pricing errors in the C-CAPM. We also show (analytically and in a Monte Carlo study) that implausible estimates of risk aversion and time preference are not puzzling in this framework and emerge as a result of rational pricing errors. While this bias essentially removes the pricing error in the traditional endowment economy, a production economy with stochastically changing investment opportunities generates large and persistent empirical pricing errors.

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1 Introduction

It is a commonly held perception that the workhorse of financial economics – the consumption-based capital asset pricing model (C-CAPM) – has fallen on hard times.[1] Most prominent is the failure to account for the U.S. equity premium with any plausible values of risk aversion, which has been referred to as the 'equity premium puzzle' (Mehra and Prescott, 1985).

The limitations of the standard C-CAPM gave rise to a vast literature of model extensions to achieve better empirical performance.[2] An open question remains, however, why leading asset pricing models fail on one particular dimension: these models have severe trouble to explain why the standard C-CAPM generates verz large empirical pricing errors (or Euler equation errors, cf. Lettau and Ludvigson, 2009, p.255),

> "Unlike the equity premium puzzle, these large Euler equation errors cannot be resolved with high values of risk aversion. To explain why the standard model fails, we need to develop [...] models that can rationalize its large pricing errors."

This paper makes three contributions. First, we show that this 'Euler equation puzzle' of Lettau and Ludvigson (2009) can be explained by the Barro-Rietz rare disaster hypothesis (Rietz, 1988; Barro, 2006, 2009), i.e., infrequent but sharp contractions (as during historical events such as the Great Depression or World War II). Hence, consumption-based models with low-probability disasters qualify as a class of models which is able to rationalize the large pricing errors of the canonical model. In fact, including low-probability disasters the standard C-CAPM not only explains the equity premium (as shown in Barro, 2006), but also can generate large Euler equation errors as found empirically in the data.

Second, we derive analytical expressions for asset returns, the stochastic discount factor (SDF), and Euler equation errors, both in an endowment economy and in a production economy with low-probability disasters. It is important to emphasize, however, that only the departure from (conditional) log-normality of asset returns, e.g., using a stochastically changing investment opportunity set as in the production economy, is able to generate sizable 'empirical' pricing errors. Our analytical results shed light on the endogenous time-varying behavior of asset returns in the (neoclassical) production economy, and the effects of rare disasters on Euler equation errors in general equilibrium.

Third, we present extensive Monte Carlo evidence to investigate the impact of low-probability disasters on the plausibility of standard C-CAPM parameter estimates. We find

[1] The consumption-based asset pricing model has its roots in the seminal articles by Rubinstein (1976), Lucas (1978), and Breeden (1979). Ludvigson (2011) provides an excellent survey of the literature.

[2] An non-exhaustive list of prominent modifications of the consumption-based model includes habit formation preferences (Campbell and Cochrane, 1999), long-run risk (Bansal and Yaron, 2004), heterogeneous agents and limited stock market participation (e.g. Guvenen, 2009).

that implausibly high empirical estimates for the risk aversion and for the time preference parameters – as typically found in the empirical literature – are not puzzling in a world with rare disaster risk and is the consequence of using the wrong pricing kernel. This finding is complementary to the result in Lettau and Ludvigson (2009, p.279).

Our analysis builds on the continuous-time formulation of dynamic stochastic general equilibrium (DGSE) models, which gives analytical tractability. We focus on Euler equation errors of Lettau and Ludvigson (2009).[3] The authors show that leading extensions of the consumption-based model – such as the long-run risk model (Bansal and Yaron, 2004), habit formation model (Campbell and Cochrane, 1999), and the limiting participation model (Guvenen, 2009) – cannot explain why the standard C-CAPM model generates large pricing errors. We now show that the extension with low-probability disasters in fact can rationalize large pricing errors of the C-CAPM.

The motivation for considering rare events as a solution to asset pricing puzzles is intuitive and goes back at least to Rietz (1988). The hypothesis has received renewed attention starting with Barro (2006), who backs up the calibration of his model by historical estimates of consumption disasters for a broad set of countries over a very long period. It has been shown recently that several asset pricing phenomena can be understood by rare disasters (Wachter, 2009; Gabaix, 2008, 2012). To the best of our knowledge, the effects of rare disasters on pricing errors have not been studied yet. In the Barro-Rietz framework, asset prices reflect risk premia for infrequent and severe disasters in which consumption drops sharply. If such rare disasters are expected by investors ex-ante and thus reflected in their consumption and investment decisions, but happen not to occur in sample, an equity premium of the magnitude observed for the U.S. data can materialize.[4] Barro shows that a calibrated version of the standard C-CAPM with rare events is able to explain the level of the U.S. equity premium at plausible parameters of risk aversion. We show that the rare disaster hypothesis helps along two other dimension, (1) explaining the empirical pricing errors, and (2) explaining implausible estimates of model parameters in empirical work.

The remainder of the paper is organized as follows. Section 2 provides a formal definition of the Euler equation errors, presents some empirical benchmark estimates and gives an intuitive preview of our main analytical and simulation-based results. Section 3 derives asset prices in an endowment economy and in a production economy with rare disasters. Section 4 derives analytical expressions for Euler equation errors in general equilibrium. Section 5

[3]As documented by a multitude of empirical studies, the standard C-CAPM leaves a substantial fraction of the average return unexplained when the model is asked to account for differences in average returns across different assets (see, e.g., empirical results in Hansen and Singleton, 1982; Lettau and Ludvigson, 2001).

[4]This is related to the statement in Cochrane (2005, p.30) that the U.S. economy and other countries with high historical equity premia may simply constitute very lucky cases of history.

contains Monte-Carlo evidence which shows that rare events model work well in explaining several dimensions of the empirical weaknesses of the standard consumption-based model including large Euler equation errors. Section 6 concludes.

2 Euler equation errors

In this section we provide a brief discussion on the definition of Euler equation errors, the empirical facts typically encountered in the data as well as a brief preview of how rare disasters may help rationalizing the empirical puzzles.

2.1 Euler equation errors and their empirical counterparts

Consider the standard first-order condition implied by the canonical version of the C-CAPM with time-separable utility functions,

$$u'(C_t) = e^{-\rho} E_t \left[u'(C_{t+1}) R_{t+1} \right], \quad u' > 0, \; u'' < 0. \tag{1}$$

The optimality condition (1) is referred to as the Euler equation. It implicitly determines the optimal path of per capita consumption C_t, given gross returns R_{t+1} on the investor's savings (or assets), and $\rho > 0$ is a subjective time-discount rate. We define the stochastic discount factor (SDF) as the process $m_s/m_t \equiv e^{-\rho(s-t)} u'(C_s)/u'(C_t)$ such that, for *any* security with price $P_{i,t}$ and instantaneous payoff $X_{i,s}$ at some future date $s \geq t$, we have

$$m_t P_{i,t} = E_t \left[m_s X_{i,s} \right] \quad \Rightarrow \quad 1 = E_t \left[(m_s/m_t) R_{i,s} \right], \tag{2}$$

where $R_{i,s} \equiv X_{i,s}/P_{i,t}$ denotes the security's return. In discrete-time models, the SDF at date $s = t+1$ is usually defined as $M_{t+1} \equiv m_{t+1}/m_t$. Hence, the Euler condition (2) can be used to discount expected payoffs on *any* asset to find their equilibrium prices: The agent is indifferent between investing into the various assets if (2) is satisfied. In this paper we study how the properties of the SDF explain pricing errors and how the SDF is determined by the general equilibrium of the economy.

Any deviations from (2) represent Euler equation errors,

$$e_R^i \;\equiv\; E_t \left[(m_s/m_t) R_{i,s} \right] - 1, \quad e_X^i \;\equiv\; E_t \left[(m_s/m_t)(R_{i,s} - R_{b,s}) \right], \tag{3}$$

based on the gross return on any tradable asset, $R_{i,s}$ or as a function of excess returns over a reference asset, $R_{i,s} - R_{b,s}$, e.g., the return on a bond (Lettau and Ludvigson, 2009). In what follows, we refer to either e_R^i or e_X^i as the Euler equation error, whereas to their empirical counterparts $\widehat{e^i_R}$ and $\widehat{e^i_X}$ as the *estimated* Euler equation error for the ith asset. The latter is defined for specific utility functions, e.g., for power utility with risk aversion γ,

$$\widehat{e^i_R} \equiv E_t\big[e^{-(s-t)\hat{\rho}}(C_s/C_t)^{-\hat{\gamma}}R_{i,s}\big] - 1, \quad \widehat{e^i_X} \equiv E_t\big[e^{-(s-t)\hat{\rho}}(C_s/C_t)^{-\hat{\gamma}}(R_{i,s} - R_{b,s})\big], \qquad (4)$$

where $\hat{\rho}$ and $\hat{\gamma}$ denote the estimated parameters of time-preference and risk aversion. These estimates are usually obtained by the generalized method of moments (GMM) of Hansen (1982) by minimizing a quadratic form of the pricing errors. The fit of the model is often expressed by the root mean squared error (RMSE), which is a summary measure of the magnitude of the fitted Euler equation errors.

2.2 Euler equation errors and empirical puzzles

As mentioned above, it is a well-established fact that the standard C-CAPM with power utility is incapable of explaining cross-sectional variation in average asset returns. In other words, the model produces substantial pricing errors (Euler equation errors) when fitted to the data. In order to obtain some benchmark estimates to be used in our theoretical section, we estimate the parameters of a standard C-CAPM pricing kernel $\beta(C_{t+1}/C_t)^{-\gamma}$ with U.S. postwar data (1947:Q2-2009:Q3). A large literature has focused on the performance of the model to simultaneously explain the return on a broad stock market portfolio and the return on a riskless asset such as the U.S. Treasury Bill (henceforth T-Bill). Moreover, it has also been shown that the C-CAPM fails to explain the return differences among stock portfolios sorted by size and book-to-market ratios (see, e.g., Lettau and Ludvigson, 2001).

To begin with, we follow Lettau and Ludvigson (2009) and estimate the C-CAPM model for two sets of test assets: First, a market portfolio, $R_{m,t}$, and the T-Bill, $R_{b,t}$, and second we add 6 size and book-to-market portfolios ($R_{FF,t}$ from Kenneth French's website).[5] We obtain the known result that the C-CAPM is seriously flawed: The parameter estimates are implausible, $\hat{\beta} = 1.5$, $\hat{\gamma} = 123.0$ for the case of two assets $R_{m,t}$ and $R_{b,t}$ (or $\hat{\beta} = 1.4$, $\hat{\gamma} = 101.6$ when including $R_{FF,t}$).[6] These estimates are grossly inconsistent with economic theory. A time discount factor above one implies that households value future consumption more than current consumption, whereas the estimated parameter of relative risk aversion is far higher than the microeconomic evidence on individuals' behavior in risky gambles.

[5] The estimation is based on gross-returns deflated by the PCE deflator. The series of consumption is obtained from the NIPA tables (real consumption of nondurables and services, expressed in per capita terms). The estimation is conducted by standard GMM with the identity matrix as a weighting matrix.

[6] Estimation on German post-war data for the two asset case (1975:Q1-2008:Q4) yields $\hat{\beta} = 0.77$, $\hat{\gamma} = 82.3$, with RMSE of 2.53%. Estimation on a longer sample from 1900 to 2008 (annual data from Global Financial Data, consumption data from Barro and Ursua, 2008) yields $\hat{\beta} = 0.64$ and $\hat{\gamma} = 6.53$, with RMSE of 1.5%. Note that the latter period includes World War I, the Great depression and the World War II episodes.

In addition, the Euler equation errors are economically large. Here, the RMSE amounts to 2.49% (3.05%) p.a. for the two-asset case (the larger cross-section), leaving a substantial fraction of the cross-sectional variation of average returns unexplained. It is puzzling to the econometrician why individuals seem to accept surprisingly large and persistent pricing errors. Economically, this result implies that consumers seem to accept a 2.5 dollar pricing error for each 100 dollar spent. As Lettau and Ludvigson (2009) further demonstrate, it is not possible to reduce the Euler equation error to smaller magnitudes (or even to zero) by choosing other parameter constellations. Additionally, they convincingly show that all the newly proposed theories of consumption-based asset pricing, as referred to before, are not capable of rationalizing the large pricing errors of the canonical model.

However, as we argue below, the consumption-based models coupled with stochastically occurring rare disasters of the Barro-Rietz type, which just happen not to occur within the sample, are able to rationalize the pricing errors of the C-CAPM *and* produce substantial biases in parameter estimates – akin to our observation in the empirical data.

2.3 Rare events and Euler equation errors - A preview

While the optimality conditions (1) and (2) are very general pricing formulas which must be fulfilled in most consumption-based models, the continuous-time formulation helps making the effects on pricing errors more explicit as the distributional assumptions directly appear. Allowing for rare disasters, suppose that a continuous-time formulation of the C-CAPM implies the following Euler equation (as shown below)

$$du'(C_t) = -(r_t^f - \rho)u'(C_t)dt - \pi_t u'(C_t)dB_t + (u'(C_t) - u'(C_{t-}))(dN_t - \lambda_t dt), \tag{5}$$

where r_t^f is the (shadow) risk-free rate, π_t is a measure of risk, B_t is a standard Brownian motion, and N_t is a standard Poisson process capturing rare events occuring at the arrival rate λ_t, and C_{t-} is the left-limit, $C_{t-} \equiv \lim_{s\to t} C_s$, for $s < t$ (cf. Merton, 1971). We obtain the SDF from the Euler equation: Use Itô's formula to rewrite (5) for $s \geq t$ as

$$d\ln u'(C_t) \;=\; -(r_t^f - \rho + \tfrac{1}{2}\pi_t^2)dt - \pi_t dB_t + (\ln u'(C_t) - \ln u'(C_{t-}))(dN_t - \lambda_t dt).$$

Now integrate and equate discounted marginal utility in s and t, $e^{-\rho(s-t)}u'(C_s)/u'(C_t)$, or

$$m_s/m_t \;\equiv\; \exp\left(-\int_t^s (r_v^f + \tfrac{1}{2}\pi_v^2)dv - \int_t^s \pi_v dB_v + \int_t^s \ln\left(\frac{u'(C_v)}{u'(C_{v-})}\right)(dN_v - \lambda_v dv)\right) \tag{6}$$

defines the *stochastic discount factor* (also known as pricing kernel or state-price density).

We are now prepared to make our two main points resolving the empirical puzzles. First, the presence of rare events can generate quite persistent pricing errors in finite samples. For illustration, consider a risk-free asset with gross return $R_{f,s} \equiv \exp(\int_t^s r_v^f dv)$. From (3) and (6), we obtain the Euler equation error, *conditioned* on no disasters as

$$\begin{aligned} e^f_{R|N_s-N_t=0} &= E_t\left[(m_s/m_t)R_{f,s}|N_s-N_t=0\right]-1 \\ &= \exp\left(-\int_t^s \ln\left(\frac{u'(C_v)}{u'(C_{v-})}\right)\lambda_v dv\right)-1<0 \quad \text{for} \quad u'(C_v)/u'(C_{v-})>1, \end{aligned}$$

which is strictly negative for consumption disasters. Our result shows that the individuals accept persistent pricing errors for the events that happen not to occur in normal times.[7] As shown in Barro (2006), rare disasters have been sufficiently frequent and large enough to explain the equity premium puzzle. As we show below analytically and by simulations, low-probability events are quantitatively important for Euler equation errors as well.

Our second main point is based on *estimated* Euler equation errors and the associated parameter estimates. Provided we have economically substantial Euler equation errors, the standard GMM procedure of obtaining parameter estimates will be severely biased.[8] As we show below in simulations, implausible high estimates of the parameter of relative risk aversion – of similar magnitudes as in empirical studies – will appear in samples where the sample frequency of rare disasters differs from their population value.

3 Asset pricing models with rare events

This section computes general equilibrium consumption and asset returns in endowment and production economies. These measures are used below to compute Euler equation errors.

3.1 Lucas' endowment economy with rare disasters

Consider a fruit-tree economy and a riskless asset in normal times but with default risk (government bond) similar to Barro (2006) using the formulation as in Posch (2011). Similar papers consider time-varying disaster probabilities (Gabaix, 2008; Wachter, 2009), which will not substantially affect our result and thus is not the focus of our analysis.

3.1.1 Description of the economy

Technology. Consider an endowment economy (Lucas, 1978). Suppose production is entirely exogenous: No resources are utilized, and there is no possibility of affecting the output of

[7]This result refers to Hansen and Jagannathan (1991, p.250), who note that the sample volatility may be substantially different than the population volatility if consumers anticipate that extremely bad events can occur with small probability when such events do not occur in the sample.

[8]In our Monte-Carlo experiments, we find that the Empirical Likelihood (EL) method of Julliard and Gosh (2008) gives even more biased estimates (not reported).

any unit at any time, $Y_t = A_t$ where A_t is the stochastic technology. Output is perishable. The law motion of A_t will be taken to follow a Markov process,

$$dA_t = \bar{\mu} A_t dt + \bar{\sigma} A_t dB_t + (\exp(\bar{\nu}) - 1) A_{t-} dN_t, \quad \bar{\nu} \in \mathbb{R}, \tag{7}$$

where B_t is a Brownian motion, and N_t is a Poisson process with arrival rate λ. The jump size is proportional to its value an instant before the jump, A_{t-}, ensuring that A_t does not jump negative. The notation A_{t-} denotes the left-limit, $A_{t-} \equiv \lim_{s \to t} A_s$, for $s < t$.

Suppose ownership of fruit-trees with productivity A_t is determined at each instant in a competitive stock market, and the production unit has outstanding one perfectly divisible equity share. A share entitles its owner to all of the unit's instantaneous output in t. Shares are traded at a competitively determined price, $P_{i,t}$. Suppose that for the risky asset,

$$dP_{i,t} = \mu P_{i,t} dt + \sigma P_{i,t} dB_t + P_{i,t-} J_t dN_t \tag{8}$$

and for a government bill with default risk

$$dP_{b,t} = P_{b,t} r dt + P_{b,t-} D_t dN_t, \quad \text{where} \quad D_t = \begin{cases} 0 & \text{with} \quad 1-q \\ \exp(\kappa) - 1 & \text{with} \quad q \end{cases} \tag{9}$$

is the default risk in case of a disaster, $\kappa < 0$ is the (degenerated) size of the default and q is the probability of default in case of a disaster (cf. Barro, 2006).

Preferences. Consider an economy with a single consumer, interpreted as a representative "stand in" for a large number of identical consumers. The consumer maximizes discounted expected life-time utility

$$U_0 \equiv E \int_0^\infty e^{-\rho t} u(C_t) dt, \quad u' > 0, \; u'' < 0. \tag{10}$$

Assuming no dividend payments, the consumer's budget constraint reads

$$dW_t = ((\mu - r) w_t W_t + r W_t - C_t)\, dt + w_t \sigma W_t dB_t + ((J_t - D_t) w_{t-} + D_t) W_{t-} dN_t, \tag{11}$$

where W_t is real financial wealth and w_t denote a consumer's share holdings.

Equilibrium properties. In this economy, it is easy to determine equilibrium quantities of consumption and asset holdings. The economy is closed and all output will be consumed, $C_t = Y_t$, and all shares will be held by capital owners.

3.1.2 Obtaining the Euler equation

Suppose that the only asset is the *market portfolio*,

$$dp_M(t) = \mu_M p_M(t) dt + \sigma_M p_M(t) dB_t - \zeta_M(t_-) p_M(t_-) dN_t, \tag{12}$$

where $\zeta_M(t)$ is considered as an exogenous stochastic jump-size, defining

$$\mu_M \equiv (\mu - r)w_t + r, \quad \sigma_M \equiv w_t\sigma, \quad \zeta_M(t) \equiv (D_t - J_t)w_t - D_t. \tag{13}$$

The consumer obtains income and has to finance its consumption stream from wealth,

$$dW_t = (\mu_M W_t - C_t)\,dt + \sigma_M W_t dB_t - \zeta_M(t_-)W_{t-}dN_t. \tag{14}$$

One can think of the original problem with budget constraint (11) as having been reduced to a simple Ramsey problem, in which we seek an optimal consumption rule given that income is generated by the uncertain yield of a (composite) asset (cf. Merton, 1973).[9]

Define the *value function* as

$$V(W_0) \equiv \max_{\{C_t\}_{t=0}^{\infty}} U_0, \quad s.t. \quad (14), \quad W_0 > 0. \tag{15}$$

Using the Bellman equation (see appendix), we obtain the *first-order condition* as

$$u'(C_t) = V_W(W_t), \tag{16}$$

for any $t \in [0, \infty)$, making consumption a function of the state variable $C_t = C(W_t)$.

It can be shown that the *Euler equation* is (cf. Posch, 2011)

$$\begin{aligned} du'(C_t) &= \big((\rho - \mu_M + \lambda)u'(C_t) - \sigma_M^2 W_t u''(C_t)C_W \\ &\quad - E^{\zeta}\left[u'(C((1-\zeta_M(t))W_t))(1-\zeta_M(t))\lambda\right]\big)dt \\ &\quad - \pi_t u'(C_t)dB_t + (u'(C((1-\zeta_M(t_-))W_{t-})) - u'(C(W_{t-})))dN_t, \end{aligned} \tag{17}$$

which implicitly determines the optimal consumption path, where the traditional market price of risk can be defined as $\pi_t \equiv -\sigma_M W_t u''(C_t)C_W/u'(C_t)$. We defined C_W as the marginal propensity to consume out of wealth, i.e., the slope of the *consumption function*.

3.1.3 General equilibrium prices

This section shows that general equilibrium conditions pin down the prices in the economy. We use the stochastic differential for consumption implied by the Euler equation (17) and the market clearing condition $C_t = A_t$ together with the exogenous dividend process (7).

Proposition 3.1 (Asset pricing) *In general equilibrium, market clearing implies*

$$\mu_M - r = -\frac{u''(C_t)C_W W_t}{u'(C(W_t))}\sigma_M^2 - \frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}\left((1-e^{\kappa})q - \zeta_M\right)\lambda \tag{18}$$

$$\sigma_M = \bar{\sigma}C_t/(C_W W_t) \tag{19}$$

$$r = \rho - \frac{u''(C_t)C_t}{u'(C_t)}\bar{\mu} - \tfrac{1}{2}\frac{u'''(C_t)C_t^2}{u'(C_t)}\bar{\sigma}^2 + \lambda - (1-(1-e^{\kappa})q)\frac{u'(e^{\bar{\nu}}C_t)}{u'(C_t)}\lambda. \tag{20}$$

[9] A more comprehensive approach considers the portfolio problem which is available on request.

as well as implicitly the portfolio jump-size

$$C((1-\zeta_M(t))W_t) \quad = \quad \exp(\bar{\nu})C(W_t). \tag{21}$$

Proof. cf. appendix ■

As a result, the higher the subjective rate of time preference, ρ, the higher is the general equilibrium interest rate to induce individuals to defer consumption (cf. Breeden, 1986). For convex marginal utility (decreasing absolute risk aversion), $u'''(c) > 0$, a lower conditional variance of dividend growth, $\bar{\sigma}^2$, and a higher conditional mean of dividend growth, $\bar{\mu}$, and a higher default probability, q, decrease the bond price and increases the interest rate.

3.1.4 Explicit solutions

As shown in Merton (1971), the standard dynamic consumption and portfolio selection problem has explicit solutions where consumption is a linear function of wealth. For later references, we provide the solution for constant relative risk aversion (CRRA).

Proposition 3.2 (CRRA preferences) *If utility exhibits constant relative risk aversion, i.e.,* $-u''(C_t)C_t/u'(C_t) = \gamma$, *then the optimal consumption function is proportional to wealth,* $C_t = C(W_t) = bW_t$, *where* $b \equiv \left(\rho + \lambda - (1-\gamma)\mu_M - (1-\zeta_M)^{1-\gamma}\lambda + (1-\gamma)\gamma\frac{1}{2}\sigma_M^2\right)/\gamma$.

Proof. see Posch (2011) ■

Corollary 3.3 *The implicit risk premium is*

$$RP = \gamma\bar{\sigma}^2 + e^{-\gamma\bar{\nu}}(1-e^{\bar{\nu}})\lambda. \tag{22}$$

whereas the disaster risk of the market premium in (18) is $e^{-\gamma\bar{\nu}}\left(1-e^{\bar{\nu}}-(1-e^{\kappa})q\right)\lambda$.

3.1.5 Stochastic discount factor

This section computes the stochastic discount factor (SDF). We obtain the SDF along the lines of (5) to (6) from the Euler equation (17), which in general equilibrium is

$$\begin{aligned} du'(C_t) \quad &= \quad (\rho - r)u'(C_t)dt + (1-e^{\kappa})u'(e^{\bar{\nu}}C_t)q\lambda dt - (u'(e^{\bar{\nu}}C_t) - u'(C_t))\lambda dt \\ &\quad\quad -\pi_t u'(C_t)dB_t + (u'(e^{\bar{\nu}}C_{t-}) - u'(C_{t-}))dN_t, \end{aligned}$$

where the deterministic term consists firstly of the difference between the subjective rate of time preference and the riskless rate, secondly a term which transforms this rate into the certainty equivalent rate of return (shadow risk-free rate), and thirdly the compensation which transforms the Poisson process to a martingale. For $s \geq t$, we obtain

$$\begin{aligned} m_s/m_t \;\equiv\; & \exp\left(-\int_t^s \left(\rho - \frac{u''(C_v)C_v}{u'(C_v)}\bar{\mu} - \tfrac{1}{2}\frac{u'''(C_v)C_v^2}{u'(C_v)}\bar{\sigma}^2 + \tfrac{1}{2}\pi^2\right) dv\right) \\ & \times \exp\left(-\int_t^s \pi_t dB_v + \int_t^s \left(\ln u'(e^{\bar{\nu}}C_{t-}) - \ln u'(C_{t-})\right) dN_v\right), \end{aligned} \tag{23}$$

as the *stochastic discount factor*, which can be used to price any asset in this economy. For the case of CRRA preferences, (23) simplifies to

$$\begin{aligned} m_s/m_t \;=\; & \exp\left(-(r - e^{-\gamma\bar{\nu}}(1-e^{\kappa})q\lambda + \tfrac{1}{2}(\gamma\bar{\sigma})^2 + (e^{-\bar{\nu}\gamma}-1)\lambda)(s-t)\right) \\ & \times \exp\left(-\gamma\bar{\sigma}(B_s - B_t) - \gamma\bar{\nu}(N_s - N_t)\right), \end{aligned} \tag{24}$$

where

$$r = \rho + \gamma\bar{\mu} - \tfrac{1}{2}\gamma(1+\gamma)\bar{\sigma}^2 + \lambda - (1-(1-e^{\kappa})q)\, e^{-\gamma\bar{\nu}}\lambda \tag{25}$$

is the (shadow) risk-free rate, or the rate of return of any zero-supply riskless security. For $\gamma < 0$ and $\kappa < 0$, the presence of rare events increases the risk-free rate of return.

3.1.6 General equilibrium consumption growth rates and asset returns

This section derives consumption growth rates and equilibrium asset returns for various financial claims. These measures are important for computing Euler equation errors.

Consumption. Consumption growth rates are exogenous in the endowment economy. Thus, consumption growth rates can be obtained from the dividend process (7),

$$\ln(C_s/C_t) \;=\; \ln(A_s/A_t) = (\bar{\mu} - \tfrac{1}{2}\bar{\sigma}^2)(s-t) + \bar{\sigma}(B_s - B_t) + \bar{\nu}(N_s - N_t). \tag{26}$$

Risky asset. Consider a claim which pays a dividend $X_{i,t+1} = A_{t+1}$, i.e., an instantaneous return in period $s = t+1$. Using the pricing kernel (24) together with (2) implies

$$\begin{aligned} R_{c,t+1} \;=\; & \exp\left(\rho + \gamma\bar{\mu} - \tfrac{1}{2}\gamma\bar{\sigma}^2 - \tfrac{1}{2}(1-\gamma)^2\bar{\sigma}^2 - (e^{(1-\gamma)\bar{\nu}} - 1)\lambda\right) \\ & \times \exp\left(\bar{\sigma}(B_{t+1} - B_t) + \bar{\nu}(N_{t+1} - N_t)\right). \end{aligned} \tag{27}$$

Riskless asset. From (24) and (2), the equilibrium asset return of any riskless security is

$$R_{f,t+1} \;=\; \exp\left(r - e^{-\gamma\bar{\nu}}(1-e^{\kappa})q\lambda\right). \tag{28}$$

whereas $R_{b,t+1} = e^{r + \int_t^{t+1} \ln(1+D_s)dN_s}$ is the equilibrium return for any riskless asset which is subject to default risk, e.g., issued exogenously by the government.

3.2 A production economy with rare events

This section obtains the pricing kernel for an economy where Euler equation errors arise as a result of rare technological improvements in a production economy (cf. Wälde, 2005). As before, an extensive discussion of the model and its solution is in Posch (2011).

3.2.1 Description of the economy

Technology. At any time, the economy has some amounts of capital, labor, and knowledge, and these are combined to produce output. The production function is a constant return to scale technology $Y_t = A_t F(K_t, L)$, where K_t is the aggregate capital stock, L is the constant population size, and A_t is the stock of knowledge or total factor productivity (TFP), which is driven by a standard Brownian motion B_t and a Poisson process $\bar{N}_t$ with arrival rate $\bar{\lambda}$,

$$dA_t = \bar{\mu} A_t dt + \bar{\sigma} A_t dB_t + (\exp(\bar{\nu}) - 1) A_{t-} d\bar{N}_t. \tag{29}$$

We introduce jumps in TFP as there is empirical evidence of Poisson jumps in output growth rates which, however, may not necessarily reflect consumption disasters (Posch, 2009).

The capital stock increases if gross investment exceeds stochastic capital depreciation,

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t + (\exp(\nu) - 1) K_{t-} dN_t, \tag{30}$$

in which Z_t is a standard Brownian motion (uncorrelated with B_t), and N_t is a Poisson process with arrival rate λ. The jump size in the capital stock is proportional and has a degenerated distribution.[10] Note that only for $\sigma = \nu = 0$, the capital stock (physical asset) is instantaneously riskless (cf. Merton, 1975).

Preferences. Consider an economy with a single consumer, interpreted as a representative "stand in" for a large number of identical consumers. The consumer maximizes expected life-time utility

$$U_0 \equiv E_0 \int_0^\infty e^{-\rho t} u(C_t) dt, \quad u' > 0, \; u'' < 0 \tag{31}$$

subject to

$$dW_t = ((r_t - \delta) W_t + w_t^L - C_t)dt + \sigma W_t dZ_t + J_t W_{t-} dN_t. \tag{32}$$

$W_t \equiv K_t / L$ denotes individual wealth, r_t is the rental rate of capital, and w_t^L is labor income. The paths of factor rewards are taken as given by the representative consumer.

[10] As in Cox, Ingersoll and Ross (1985, p.366), individuals can invest in physical production indirectly through firms or directly, in effect creating their own firms. There is a market for instantaneous borrowing and lending at the interest rate $r_t = Y_K$, which is determined as part of the competitive equilibrium of the economy. There are markets for contingent claims which are all zero-supply assets in equilibrium.

Equilibrium properties. In equilibrium, factors of production are rewarded with value marginal products, $r_t = Y_K$ and $w_t^L = Y_L$. The goods market clearing condition demands

$$Y_t = C_t + I_t. \tag{33}$$

Solving the model requires the aggregate capital accumulation constraint (30), the goods market equilibrium (33), equilibrium factor rewards of perfectly competitive firms, and the first-order condition for consumption. It is a system of stochastic differential equations determining, given initial conditions, the paths of K_t, Y_t, r_t, w_t^L and C_t, respectively.

3.2.2 Obtaining the Euler equation

Define the *value function* as

$$V(W_0, A_0) = \max_{\{C_t\}_{t=0}^{\infty}} U_0 \quad s.t. \quad (32) \quad \text{and} \quad (29), \tag{34}$$

denoting the present value of expected utility along the optimal program. Using the Bellman equation similar to the endowment economy, we obtain the *first-order condition* as

$$u'(C_t) = V_W(W_t, A_t), \tag{35}$$

for any $t \in [0, \infty)$, making consumption a function of the state variables $C_t = C(W_t, A_t)$.

It can be shown that the *Euler equation* is (cf. appendix)

$$\begin{aligned} du'(C_t) &= (\rho - (r_t - \delta) + \lambda + \bar{\lambda})u'(C_t)dt - u'(C(e^{\nu}W_t, A_t))e^{\nu}\lambda dt - u'(C(W_t, e^{\bar{\nu}}A_t))\bar{\lambda} \\ &\quad -\sigma^2 u''(C_t)C_W W_t dt + u''(C_t)(C_A A_t \bar{\sigma} dB_t + C_W W_t \sigma dZ_t) \\ &\quad +[u'(C(W_{t-}, e^{\bar{\nu}}A_{t-})) - u'(C(W_{t-}, A_{t-}))]d\bar{N}_t \\ &\quad +[u'(C(e^{\nu}W_{t-}, A_{t-})) - u'(C(W_{t-}, A_{t-}))]dN_t, \end{aligned} \tag{36}$$

which implicitly determines the optimal consumption path. Comparing to the Euler equation in the endowment economy (17), the stochastic discount factor implied by (36) now has richer dynamics with time-varying interest rates and two sources of low-probability events.

3.2.3 General equilibrium prices

Note that physical capital is the only asset that is held in equilibrium, henceforth the market portfolio. Since all other assets are zero-supply assets, we can price any financial claim as if they were traded assets using the stochastic discount factor.

3.2.4 Explicit solutions

A convenient way to describe the behavior of the economy is in terms of the evolution of C_t, A_t and W_t. Similar to the endowment economy there are explicit solutions available, due to the non-linearities only for specific parameter restrictions. Below we use two known restrictions where the *policy function* $C_t = C(A_t, W_t)$ (or consumption function) is available, and many economic variables can be solved in closed form.

Proposition 3.4 (linear-policy-function) *If the production function is Cobb-Douglas, $Y_t = A_t K_t^{\alpha} L^{1-\alpha}$, utility exhibits constant relative risk aversion, i.e., $-u''(C_t)C_t/u'(C_t) = \gamma$, and $\alpha = \gamma$, then optimal consumption is linear in wealth.*

$$\alpha = \gamma \quad \Rightarrow \quad C_t = C(W_t) = \phi W_t$$
$$\textit{where} \quad \phi \equiv (\rho - (e^{(1-\gamma)\nu} - 1)\lambda + (1-\gamma)\delta)/\gamma + \tfrac{1}{2}(1-\gamma)\sigma^2 \tag{37}$$

Proof. see appendix ■

Corollary 3.5 *The implicit risk premium is*

$$RP|_{\alpha=\gamma} = \gamma\sigma^2 + e^{-\gamma\nu}(1-e^{\nu})\lambda. \tag{38}$$

Proposition 3.6 (constant-saving-function) *If the production function is Cobb-Douglas, $Y_t = A_t K_t^{\alpha} L^{1-\alpha}$, utility exhibits constant relative risk aversion, i.e., $-u''(C_t)C_t/u'(C_t) = \gamma$, and the subjective discount factor is*

$$\bar{\rho} \equiv (e^{-\theta\bar{\nu}} - 1)\bar{\lambda} + (e^{(1-\alpha\gamma)\nu} - 1)\lambda - \gamma\bar{\mu} + \tfrac{1}{2}\left(\gamma(1+\gamma)\bar{\sigma}^2 - \alpha\gamma(1-\alpha\gamma)\sigma^2\right) - (1-\alpha\gamma)\delta,$$

then optimal consumption is proportional to current income (i.e., non-linear in wealth).

$$\rho = \bar{\rho} \quad \Rightarrow \quad C_t = C(W_t, A_t) = (1-s)A_t W_t^{\alpha}, \quad \gamma > 1, \quad \textit{where} \quad s \equiv 1/\gamma \tag{39}$$

Proof. see appendix ■

Corollary 3.7 *The implicit risk premium is*

$$RP|_{\rho=\bar{\rho}} = \alpha\gamma\sigma^2 + e^{-\alpha\gamma\nu}(1-e^{\nu})\lambda. \tag{40}$$

It is interesting to note that the market premium (or implicit risk premium) does not reward the risk associated with a stochastic TFP process. The intuitive reason is that at the aggregate level all contingent claims are in zero supply. Hence, the only asset that affects the intertemporal investment opportunities of the market is the physical asset.

3.2.5 Stochastic discount factor

Similar to the endowment economy, the SDF is obtained along the lines of (5) to (6) from the Euler equation (36). For $s \geq t$, we obtain

$$
\begin{aligned}
m_s/m_t = \exp\Bigg(- \int_t^s \left(r_l - \delta - \lambda - \bar{\lambda} + \frac{u'(C(e^{\nu}W_l, A_l))}{u'(C(W_l, A_l))} e^{\nu}\lambda + \frac{u'(C(W_l, e^{\bar{\nu}}A_l))}{u'(C(W_l, A_l))} \bar{\lambda} \right) dl \\
+ \frac{u''(C_l) C_W W_l}{u'(C_l)} \sigma^2 dl - \tfrac{1}{2} \int_t^s \frac{(u''(C_l))^2}{(u'(C_l))^2} ((C_A A_l \bar{\sigma})^2 + (C_W W_l \sigma)^2) dl \\
+ \int_t^s \frac{u''(C_l)}{u'(C_l)} (C_A A_l \bar{\sigma} dB_l + C_W W_l \sigma dZ_l) \\
+ \int_t^s \ln\left(\frac{u'(C(e^{\nu}W_{l-}, A_{l-}))}{u'(C(W_{l-}, A_{l-}))} \right) dN_l + \int_t^s \ln\left(\frac{u'(C(W_{l-}, e^{\bar{\nu}}A_{l-}))}{u'(C(W_{l-}, A_{l-}))} \right) d\bar{N}_l \Bigg)
\end{aligned}
$$

as the *stochastic discount factor*, which can be used to price any asset in this economy. For the case of CRRA preferences we obtain for our closed-form solutions,

$$
\begin{aligned}
m_s/m_t|_{\alpha=\gamma} &= \exp\left(- \int_t^s (r_l - \delta) dl + [\lambda - e^{(1-\gamma)\nu}\lambda + \gamma\sigma^2 - \tfrac{1}{2}(\gamma\sigma)^2](s-t) \right) \\
&\quad \times \exp\left(-\gamma\sigma(Z_s - Z_t) - \gamma\nu(N_s - N_t) \right), \qquad (41) \\
m_s/m_t|_{\rho=\bar{\rho}} &= \exp\left(- \int_t^s (r_l - \delta) dl + [(1 - e^{(1-\alpha\gamma)\nu})\lambda + (1 - e^{-\gamma\bar{\nu}})\bar{\lambda}](s-t) \right) \\
&\quad \times \exp\left([\gamma\alpha\sigma^2 - \tfrac{1}{2}(\gamma\bar{\sigma})^2 - \tfrac{1}{2}(\alpha\gamma\sigma)^2](s-t) - \gamma\bar{\sigma}(B_s - B_t) \right) \\
&\quad \times \exp\left(-\alpha\gamma\sigma(Z_s - Z_t) - \alpha\gamma\nu(N_s - N_t) - \gamma\bar{\nu}(\bar{N}_s - \bar{N}_t) \right). \qquad (42)
\end{aligned}
$$

In the general case, the implicit risk premium will be time-varying and asymmetric over the business cycle (Posch, 2011). This also implies that the SDF is no longer available in closed form, which complicates any Monte Carlo study without generating new insights.

3.2.6 General equilibrium consumption growth rates and asset returns

This section derives consumption growth rates and equilibrium asset returns for various financial claims. We focus on two parametric restrictions under which consumption, the pricing kernel, as well as asset returns on various claims are available in closed form. This strategy greatly simplifies our effort later to compute Euler equation errors.

Consumption. In contrast to the endowment economy, the (neoclassical) production economy introduces transitional dynamics, which imply that consumption growth rates, at least transitionally, are endogenous and will depend on the specific solution.

Given the closed-form solutions as of Propositions 3.4 and 3.6, it is straightforward to obtain consumption growth rates (cf. appendix),

$$\ln(C_s/C_t)|_{\alpha=\gamma} = 1/\alpha \int_t^s r_v dv - (\phi+\delta+\tfrac{1}{2}\sigma^2)(s-t) + \sigma(Z_s - Z_t) + \nu(N_s - N_t). \quad (43)$$

$$\begin{aligned}\ln(C_s/C_t)|_{\rho=\bar{\rho}} &= 1/\gamma \int_t^s r_v dv + (\bar{\mu} - \tfrac{1}{2}\bar{\sigma}^2 - \alpha\delta - \tfrac{1}{2}\alpha\sigma^2)(s-t) + \bar{\sigma}(B_s - B_t) \\ &\quad + \alpha\sigma(Z_s - Z_t) + \alpha\nu(N_s - N_t) + \bar{\nu}(\bar{N}_s - \bar{N}_t). \quad (44)\end{aligned}$$

Risky assets. Asset prices in this economy are driven by the rental rate of physical capital. For our parametric restriction, we obtain these capital rewards in closed-form. As shown in the appendix, the dynamics for the rental rate of capital are given by

$$\begin{aligned}dr_t &= c_1\big(c_2 - r_t\big)r_t dt + (\alpha-1)\sigma r_t dZ_t + \bar{\sigma} r_t dB_t + (\exp((\alpha-1)\nu)-1)r_{t-}dN_t \\ &\quad + (\exp(\bar{\nu})-1)r_{t-}d\bar{N}_t. \quad (45)\end{aligned}$$

This result is remarkable as it implies a specific structure for a tendency of this rate towards some equilibrium value c_2 at the speed of reversion of c_1 (cf. Posch, 2009),

$$c_1|_{\alpha=\gamma} \equiv \tfrac{1-\alpha}{\alpha}, \quad c_2|_{\alpha=\gamma} \equiv \alpha\phi + \alpha\delta - \tfrac{1}{2}\alpha(\alpha-2)\sigma^2 - \tfrac{\alpha}{\alpha-1}\bar{\mu},$$
$$c_1|_{\rho=\bar{\rho}} \equiv \tfrac{1-\alpha}{\alpha\gamma}, \quad c_2|_{\rho=\bar{\rho}} \equiv \alpha\gamma\delta - \tfrac{1}{2}\alpha\gamma(\alpha-2)\sigma^2 - \tfrac{\alpha\gamma}{\alpha-1}\bar{\mu}.$$

Consider a risky bond that pays continuously at the rate, r_t. Investing into this asset gives the random dividend process $X_{b,t+1} = e^{\int_t^{t+1} r_s ds}$. Using the pricing kernels (41) or (42) together with (2) implies

$$R_{b,t+1}|_{\alpha=\gamma} = \exp\left(\int_t^{t+1}(r_s-\delta)ds - (\gamma\sigma^2 + e^{-\gamma\nu}(1-e^{\nu})\lambda)\right), \quad (46)$$

$$R_{b,t+1}|_{\rho=\bar{\rho}} = \exp\left(\int_t^{t+1}(r_s-\delta)ds - (\gamma\alpha\sigma^2 + e^{-\alpha\gamma\nu}(1-e^{\nu})\lambda)\right). \quad (47)$$

Consider a claim on *output* which pays $X_{c,t+1} = A_{t+1}K_{t+1}^{\alpha}$, i.e., an instantaneous return in period $s = t+1$. As shown in the appendix, for the case of $\alpha = \gamma$, we obtain a closed-form expression for the asset's return,

$$\begin{aligned}R_{c,t+1}|_{\alpha=\gamma} &= \exp\left(\int_t^{t+1}(r_s-\delta)ds - \tfrac{1}{2}\bar{\sigma}^2 - \lambda + e^{(1-\gamma)\nu}\lambda - \gamma\sigma^2 + \tfrac{1}{2}(\gamma\sigma)^2 - (e^{\bar{\nu}}-1)\bar{\lambda}\right) \\ &\quad \times \exp\big(\bar{\sigma}(B_{t+1}-B_t) + \alpha\sigma(Z_{t+1}-Z_t) + \alpha\nu(N_{t+1}-N_t) + \bar{\nu}(\bar{N}_{t+1}-\bar{N}_t)\big), \quad (48)\end{aligned}$$

Similarly, consider a claim on *capital*, which pays $X_{c,t+1} = K_{t+1}^{\alpha\gamma}$ at date $s = t+1$. This particular function has been chosen in order to get a closed-form expression in the case where $\rho = \bar{\rho}$, which turns out to be

$$\begin{aligned}R_{c,t+1}|_{\rho=\bar{\rho}} &= \exp\left(\int_t^{t+1}(r_s-\delta)ds - \lambda + e^{(1-\alpha\gamma)\nu}\lambda - \gamma\alpha\sigma^2 + \tfrac{1}{2}(\alpha\gamma\sigma)^2\right) \\ &\quad \times \exp\left(\alpha\gamma\sigma(Z_{t+1}-Z_t) + \alpha\gamma\nu(N_{t+1}-N_t)\right). \quad (49)\end{aligned}$$

In what follows, the equilibrium consumption growth rates and asset returns are employed to compute Euler equation errors in the production economy.

4 Results

This section computes Euler equation (EE) errors in endowment and production economies. It shows that the Barro-Rietz 'rare disaster hypothesis' generates large pricing errors. It also shows that the standard approach of estimating the parameters of relative risk aversion and time preference is severely affected in samples where the rare events anticipated by consumers do not occur (cf. Hansen and Jagannathan, 1991, p.250).

4.1 Euler equation errors in finite samples

We illustrate the approach of computing EE errors using the endowment economy. A similar approach is applicable to production economies using our closed-form solutions. Consider two assets, i.e., the government bill, $R_{b,t+1}$, and the claim on dividends, $R_{c,t+1}$.

From the definition of EE errors (3), for any asset i and CRRA preferences

$$e_R^i = E_t\left[e^{-\rho-\gamma\bar{\mu}+\frac{1}{2}\gamma\bar{\sigma}^2-\gamma\bar{\sigma}(B_{t+1}-B_t)-\gamma\bar{\nu}(N_{t+1}-N_t)}R_{i,t+1}\right]-1, \tag{50}$$

where we inserted the SDF from (24) and the (shadow) risk-free rate (25). Note that EE errors based on excess returns can be obtained from $e_X^i = e_R^i - e_R^b$ for any asset i.

Risky asset. Inserting the one-period equilibrium return on the risky asset gives

$$e_R^c = E_t\left[e^{-\frac{1}{2}(1-\gamma)^2\bar{\sigma}^2-(e^{(1-\gamma)\bar{\nu}}-1)\lambda+(1-\gamma)\bar{\sigma}(B_{t+1}-B_t)+(1-\gamma)\bar{\nu}(N_{t+1}-N_t)}\right]-1.$$

Conditional on no disasters, on average we can rationalize EE errors

$$\begin{aligned} e^c_{R|N_{t+1}-N_t=0} &= E_t\left[e^{-\frac{1}{2}(1-\gamma)^2\bar{\sigma}^2-(e^{(1-\gamma)\bar{\nu}}-1)\lambda+(1-\gamma)\bar{\sigma}(B_{t+1}-B_t)}\right]-1 \\ &= \exp\left(-(e^{(1-\gamma)\bar{\nu}}-1)\lambda\right)-1. \end{aligned} \tag{51}$$

For Barro's calibration of $\lambda = 0.017$, $\bar{\nu} = -0.4$, the absolute EE error is about 3.9% for $\gamma = 4$ and further increases with risk aversion. Hence, we argue that the EE error can be large in finite samples. We therefore cannot rule out that empirical pricing errors measure disaster risk, as the probability of no disaster occurring in a randomly selected sample of $T = 50$ years is $p(N_{t+T} - N_t = 0) = e^{-\lambda T} = 43\%$.

Riskless asset. Inserting the one-period equilibrium returns on the government bill and the truly riskless asset ($q = 0$), we obtain EE errors

$$\begin{aligned} e_R^b &= E_t\left[e^{e^{-\gamma\bar{\nu}}(1-e^{\kappa})q\lambda-(e^{-\bar{\nu}\gamma}-1)\lambda-\frac{1}{2}(\gamma\bar{\sigma})^2-\gamma\bar{\sigma}(B_{t+1}-B_t)-\gamma\bar{\nu}(N_{t+1}-N_t)+\int_t^{t+1}\ln(1+D_s)dN_s}\right]-1, \\ e_R^f &= E_t\left[e^{-(\frac{1}{2}(\gamma\bar{\sigma})^2+(e^{-\bar{\nu}\gamma}-1)\lambda)-\gamma\bar{\sigma}(B_{t+1}-B_t)-\gamma\bar{\nu}(N_{t+1}-N_t)}\right]-1. \end{aligned}$$

Conditional on no disasters, on average we can rationalize EE errors

$$e^b_{R|N_{t+1}-N_t=0} = \exp\left(-(e^{-\bar{\nu}\gamma}-1)\lambda + e^{-\gamma\bar{\nu}}(1-e^{\kappa})q\lambda\right) - 1, \quad (52)$$

$$e^f_{R|N_{t+1}-N_t=0} = \exp\left(-(e^{-\bar{\nu}\gamma}-1)\lambda\right) - 1. \quad (53)$$

Obviously, the presence of default risk reduces the EE error in quiet times for that particular asset. Neither the disaster nor the default occurred in the sample.

For the production economy, we would obtain for the risky bond (cf. appendix)

$$e^b_{R|N_{t+1}-N_t=0}|_{\alpha=\gamma} = \exp\left((1-e^{-\nu\gamma})\lambda\right) - 1, \quad (54)$$

$$e^b_{R|N_{t+1}-N_t=0}|_{\bar{\rho}=\rho} = \exp\left((1-e^{-\alpha\gamma\nu})\lambda\right) - 1. \quad (55)$$

The claims on capital and output do not generate persistent pricing errors as long as there are no rare events in total factor productivity (as in Wälde, 2005). In such cases,

$$e^c_{R|\bar{N}_{t+1}-\bar{N}_t=0}|_{\alpha=\gamma} = \exp\left((1-e^{\bar{\nu}})\bar{\lambda}\right) - 1,$$

$$e^c_{R|\bar{N}_{t+1}-\bar{N}_t=0}|_{\rho=\bar{\rho}} = \exp\left((1-e^{-\gamma\bar{\nu}})\bar{\lambda}\right) - 1.$$

Two remarks are noteworthy. First, the particular assets were chosen to obtain analytical expressions for EE errors. In general – conditional on no disasters – claims on assets or technology may also produce substantial pricing errors. Second, based on excess returns we may rationalize EE errors for the claims on capital and output, $e^c_X = e^c_R - e^b_R$.

Finally, the root mean square error (RMSE) is the average Euler equation errors across both assets – in our case the excess return and the bill return – and observation periods,

$$RMSE = \left(\frac{1}{T}\sum_{t=1}^{T}\left[\tfrac{1}{2}(e^c_{X,t})^2 + \tfrac{1}{2}(e^b_{R,t})^2\right]\right)^{\frac{1}{2}}. \quad (56)$$

We may interpret the RMSE as the average pricing error based on the two assets.

4.2 Estimated Euler equation errors

We illustrate the implications for the *estimated* Euler equation errors for the endowment economy. A similar derivation can be conducted out for the production economy. Consider the government bill, $R_{b,t+1}$, and the claim on dividends, $R_{c,t+1}$.

Using estimated EE errors in (4), for any asset i and CRRA preferences

$$\widehat{e^i_R} = E_t\left[e^{-\hat{\rho}-\hat{\gamma}\bar{\mu}+\frac{1}{2}\hat{\gamma}\bar{\sigma}^2-\hat{\gamma}\bar{\sigma}(B_{t+1}-B_t)-\hat{\gamma}\bar{\nu}(N_{t+1}-N_t)}R_{i,t+1}\right] - 1,$$

where we inserted the equilibrium consumption growth rate from (26). The estimated EE errors for excess returns can be obtained from $\widehat{e^i_X} = \widehat{e^i_R} - \widehat{e^b_R}$ for any asset i.

Risky asset. Conditional on no disasters, the estimated EE errors for the one-period equilibrium return on the risky claim are

$$\begin{aligned}
\widehat{e^c_{R|N_{t+1}-N_t=0}} &= E_t\left[e^{\rho-\hat{\rho}+(\gamma-\hat{\gamma})(\bar{\mu}-\frac{1}{2}\bar{\sigma}^2)-\frac{1}{2}(1-\gamma)^2\bar{\sigma}^2-(e^{(1-\gamma)\bar{\nu}}-1)\lambda+(1-\hat{\gamma})\bar{\sigma}(B_{t+1}-B_t)}\right]-1 \\
&= \exp\left(\rho-\hat{\rho}+(\gamma-\hat{\gamma})(\bar{\mu}-\tfrac{1}{2}\bar{\sigma}^2)-\tfrac{1}{2}((1-\gamma)^2-(1-\hat{\gamma})^2)\bar{\sigma}^2\right) \\
&\quad \times\exp\left(-(e^{(1-\gamma)\bar{\nu}}-1)\lambda\right)-1. \qquad (57)
\end{aligned}$$

The result in (57) clearly shows that in order to minimize the estimated EE errors, the parameter estimates are biased as long as $(e^{(1-\gamma)\bar{\nu}}-1)\lambda \neq 0$.

Riskless asset. Conditional on no disasters, the estimated EE errors for the one-period equilibrium return the government bill are

$$\begin{aligned}
\widehat{e^b_{R|N_{t+1}-N_t=0}} &= E_t\left[e^{r-\hat{r}-\frac{1}{2}(\hat{\gamma}\bar{\sigma})^2-\hat{\gamma}\bar{\sigma}(B_{t+1}-B_t)}\right]-1 \\
&= \exp\left(\rho-\hat{\rho}+(\gamma-\hat{\gamma})(\bar{\mu}-\tfrac{1}{2}\bar{\sigma}^2)-(\gamma^2-\hat{\gamma}^2)\tfrac{1}{2}\bar{\sigma}^2\right) \\
&\quad \times\exp\left(-((1-(1-e^{\kappa})q)\,e^{-\gamma\bar{\nu}}-1)\lambda\right)-1, \qquad (58)
\end{aligned}$$

where $\hat{r} \equiv \hat{\rho}+\hat{\gamma}\bar{\mu}-\frac{1}{2}\hat{\gamma}(1+\hat{\gamma})\bar{\sigma}^2$ is the risk-free rate in a world in which rare disasters happen not to occur. In fact, the effect of rare disasters on the equilibrium risk-free rate, $\lambda-(1-(1-e^{\kappa})q)\,e^{-\gamma\bar{\nu}}\lambda$, will be captured by $r-\hat{r}$ through biased estimates of ρ and γ.

4.3 The performance of the C-CAPM

In the standard C-CAPM estimation, GMM chooses $\hat{\rho}$ and $\hat{\gamma}$ such as to minimize the EE errors across assets (e.g., Lettau and Ludvigson, 2009). In particular, we encounter the square root of the average EE errors for the tth observation,

$$\widehat{RMSE}_t = \sqrt{\tfrac{1}{2}\left(\widehat{e^c_{X,t}}\right)^2+\tfrac{1}{2}\left(\widehat{e^b_{R,t}}\right)^2}.$$

Consider now the case of rare disasters, i.e., $\bar{\nu}<0$ (and $\kappa<0$). Now the EE error for the risky claim (conditional on no disasters) in (51) on average is positive for $\gamma<1$, whereas negative for $\gamma>1$. Further, for the government bill (conditional on no disasters) in (52) on average is unambiguously negative as the risk free rate is biased downwards. Therefore, the minimization procedure tends $\hat{\gamma}$ and $\hat{\rho}$ towards values such that they increase $\hat{r}$ in (58), taking account for the effects on the estimated EE errors in (57).[11]

[11] In order to minimize (57) and (58), both equations should hold simultaneously,

$$\begin{aligned}
\rho-\hat{\rho}+(\gamma-\hat{\gamma})(\bar{\mu}-\tfrac{1}{2}\bar{\sigma}^2)-\tfrac{1}{2}((1-\gamma)^2-(1-\hat{\gamma})^2)\bar{\sigma}^2 &\approx (e^{(1-\gamma)\bar{\nu}}-1)\lambda, \\
\rho-\hat{\rho}+(\gamma-\hat{\gamma})(\bar{\mu}-\tfrac{1}{2}\bar{\sigma}^2)-(\gamma^2-\hat{\gamma}^2)\tfrac{1}{2}\bar{\sigma}^2 &\approx ((1-(1-e^{\kappa})q)\,e^{-\gamma\bar{\nu}}-1)\lambda.
\end{aligned}$$

Hence, in order to minimize Euler equation errors, the GMM procedure tends to bias both $\hat{\rho}$ and $\hat{\gamma}$.

5 Monte Carlo experiments

In this section we provide Monte Carlo evidence on the impact of low-probability events on the performance of the consumption-based asset pricing model. In particular, we seek a better understanding of how the estimated parameters and pricing errors are affected by the presence of rare disasters. In what follows we describe the general setup of this analysis and report our simulation results.

5.1 The simulation approach

We first simulate equilibrium asset prices and consumption paths from the parameterized consumption-based models with rare events (cf. Tables 1 and 2).[12] We consider both the endowment economy and the production economy for which we derive analytical expressions for asset prices. Consistent with the sample size in empirical studies of the C-CAPM, the simulated sample paths have a length of 50 years using 5,000 Monte Carlo draws.[13]

The idea is to study whether we are able to reproduce the empirical failure of the C-CAPM in simulated data generated from economies which are infrequently hit by rare disasters. We consider a standard power utility C-CAPM pricing kernel whose parameters an econometrician would estimate when he or she is confronted with artificial data similar to Lettau and Ludvigson (2009). We are mainly interested whether the estimated C-CAPM generates EE errors using the simulated series. In other words, does a wrongly specified pricing kernel – despite biased estimates for the time preference and risk aversion parameter – generate EE errors or not? This is of particular interest since leading asset pricing models such as the long-run risk model (Bansal and Yaron, 2004), the habit formation model (Campbell and Cochrane, 1999), and the limited participation model (Guvenen, 2009) fail to produce substantial pricing errors. As such, these theories are not able to explain one important dimension of failure of the canonical model. Moreover, we shed light on the performance of C-CAPM (regarding the bias and plausibility of estimated structural parameters) in the presence of rare events, more generally.

5.2 The simulation results

In Tables 4 to 10 we show the results from the Monte-Carlo simulations (cf. Appendix A). Our main quantities of interest are the average pricing error ($RMSE$), i.e., the rational EE errors, the empirical pricing errors ($\widehat{RMSE}$), i.e., a measure of estimated EE errors, and the

[12]The parameterization follows the literature on rare disasters in endowment and production economies (see e.g., Barro, 2009, Wachter, 2009, and Posch, 2009).

[13]The simulated data is sampled at quarterly frequency (e.g., Lettau and Ludvigson, 2001).

Table 1: Parameterization (endowment economy)

		(1)	(2)	(3)	(4)	(5)
ρ	rate of time preference	0.03	**0.03**	0.03	0.03	0.03
γ	coef. of relative risk aversion	0.5	**4**	4	4	4
$\bar{\mu}$	consumption growth	0.01	**0.01**	0.01	0.01	0.01
$\bar{\sigma}$	consumption noise	0.005	**0.005**	0.005	0.005	0.005
$-\bar{\nu}$	size of consumption disaster	0.4	**0.4**	0.55	0.4	0
$\bar{\lambda}$	consumption disaster probability	0.017	**0.017**	0.017	0.017	0
$-\kappa$	size of government default	0	**0**	0	0.3	0
q	default probability	0	**0**	0	0.5	0

Table 2: Parameterization (production economy)

		(1)	(2)	(3)	(4)	(5)
ρ	rate of time preference	0.03	**0.024**	0.017	0.016	0.03
γ	coef. of relative risk aversion	0.5	**4**	4	4	4
α	output elasticity of capital	0.5	**0.6**	0.6	0.6	0.6
δ	capital depreciation	0.025	**0.025**	0.025	0.025	0.05
$\bar{\mu}$	productivity growth	0.02	**0.01**	0.01	0.01	0.01
$\bar{\sigma}$	productivity noise	0.01	**0.01**	0.01	0.01	0.01
$-\bar{\nu}$	size of productivity slump	0.01	**0.01**	0.01	0	0
$\bar{\lambda}$	productivity jump probability	0.2	**0.2**	0.2	0	0
σ	capital stochastic depreciation	0.005	**0.005**	0.005	0.005	0.005
$-\nu$	size of capital disaster	0.55	**0.55**	0.4	0.55	0
λ	capital disaster probability	0.017	**0.017**	0.017	0.017	0

respective structural parameter estimates $\hat{\beta}$ and $\hat{\gamma}$ obtained by estimating a standard power utility C-CAPM to the simulated data. In addition, we report the distributional properties of asset returns, equity premium, and consumption growth (in annualized percentage terms). For reading convenience, we report results *conditional* on the case of no disasters, i.e., only for those cases in which no disaster happened to occur over the 50 year period even though such diasters were expected by the consumers ex-ante (also known as 'Peso problem').[14] Conditioning on no disasters case may be regarded as similar to studying a sample period such as the post-war period in the U.S. without major consumption disasters (Barro, 2006). Those simulations, however, may include more frequent low-probability events such as 'smaller' jumps in productivity similar to Posch (2009). As benchmark cases, Tables 6 and 11 report the estimation results for the standard C-CAPM model without Poisson uncertainty.

Our results support our claim that rare events can have a strong impact on the estimates of structural parameters and pricing errors. For example, the endowment economy in Table 4 on average generates parameter estimates of $\hat{\beta} = 1.6$ and $\hat{\gamma} = 282.4$ as compared to $\hat{\beta} = 1.5$ and $\hat{\gamma} = 123.0$ in the data. While we do not find (estimated) pricing errors of the C-CAPM in the endowment economy, we find substantial pricing errors in the production economy. Our results in Tables 8 to 10 show that the C-CAPM (with power utility) generates large pricing errors on average between 1% and 3% (indicated by $\widehat{RMSE}$ in the tables), of similar size as 2.5% in the data. Thus, departures from log-normality in the cases where we conditioned on no disasters, e.g., through a stochastically changing investment opportunities in the production economy, seem to be important to generate estimated EE errors.

Our economies with rare events are calibrated to very reasonable values of 4 for the coefficient of relative risk-aversion and 0.97 for the subjective time discount factor.[15] Yet, if anticipated consumption disasters do not occur in sample, we observe severely biased and implausible parameter estimates that are well-known from empirical results in the literature. Our simulation-based evidence therefore suggests that such biased and implausible parameter estimates of structural preference parameters of the C-CAPM are not surprising in a world where agents are concerned about rare negative consumption shocks.

To summarize, unlike models of habit formation and/or long-run risk we refer to in the introduction, models incorporating rare disasters are able to solve the EE puzzle by Lettau and Ludvigson (2009). At the same time, we find a severe bias in parameter estimates of the subjective time discount factor and the coefficient of relative risk aversion (RRA) for cases in which consumption disasters do not occur in the sample. Our results suggest that

[14] A separate Referee's appendix has unconditional simulation results and is available on request.

[15] We encounter problems of convergence of the GMM procedure for the case of extreme values for the disaster size and the parameter of relative risk aversion (e.g., parameterization (3) in Table 1). To avoid that pricing errors simply show convergence problems, these cases are discarded in the Monte Carlo study.

the Barro-Rietz rare disaster hypothesis together with a stochastically changing investment opportunity set is able to account for the empirical failures of the C-CAPM.

6 Conclusion

In this paper we study the impact of rare disasters (such as wars or natural catastrophes) on Euler equation (EE) errors and the empirical performance of the consumption-based asset pricing model in general. For this purpose, we derive analytical asset pricing implications and EE errors both in an endowment as well as a production economy with stochastically occurring disasters. In extensive simulations we also investigate the impact of rare disasters on estimates of structural parameters of the consumption-based model and the empirical performance of the model. Thus, our paper seeks to provide a better understanding of why the standard model fails so dramatically when fitted to the data.

Allowing for low-probability events in an otherwise standard C-CAPM explains why the canonical model generates large and persistent pricing errors when confronted by the data. Hence, consumption-based models with rare disasters (Barro, 2006; Gabaix, 2008, 2012) qualify as a class of models which rationalize the Euler equations puzzle of Lettau and Ludvigson (2009). We show analytically and through simulations based on realistic calibrations that the poor empirical performance and implausible estimates of risk aversion and time preference are not puzzling in a world with rare disasters. Our results therefore suggest that the Barro-Rietz rare disaster hypothesis together with a stochastically changing investment opportunity set is able to account for one of the most puzzling empirical failure of the C-CAPM, i.e., the large and persistent empirical pricing errors.

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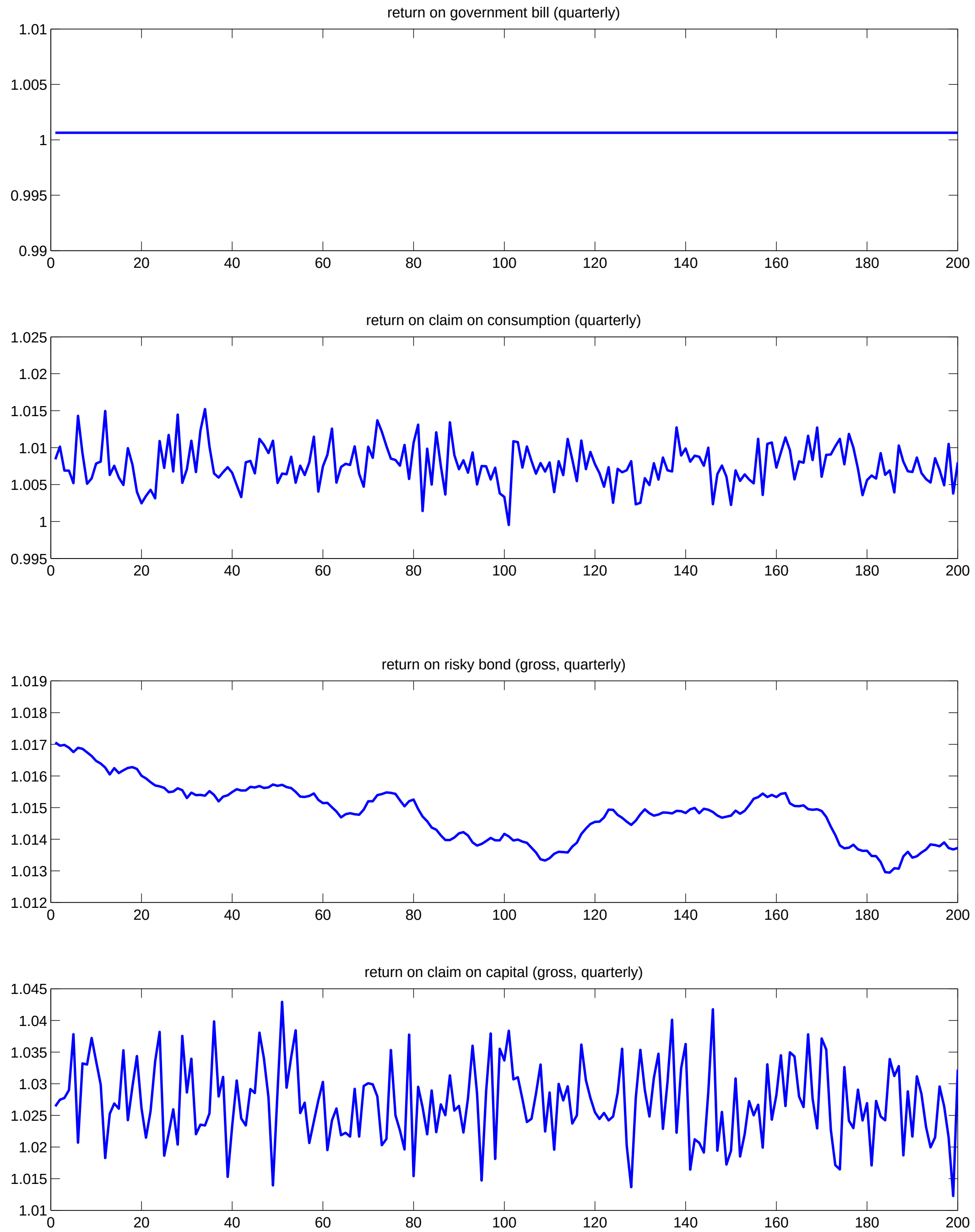
A Tables and Figures

Figure 1: General equilibrium asset returns



Notes: This figure illustrates the equilibrium asset returns and shows one realization of the return to the relatively riskless asset and the risky asset in the endowment economy (upper two panels, parameterization (2) in Table 1) and the production economy (lower two panels, parameterization (2) in Table 2), respectively.

Table 3: C-CAPM simulation results (endowment economy)

Results	analytical solution	conditional (no disasters)			
	parameterization (1)	Mean	Std. dev.	Mode	Median
β	factor of time preference	0.97			
γ	coef. of relative risk aversion	0.50			
e_R^b	EE error risky bond	−0.38	0.04	−0.39	−0.38
e_X^c	EE error excess return	0.68	0.07	0.65	0.68
$RMSE$	root mean square error	0.55	0.06	0.52	0.55
Observed random variables					
$R_{b,t+1}$	bill return	3.13	0.00	3.00	3.13
$R_{c,t+1}$	equity return	3.83	0.07	3.78	3.83
$R_{c,t+1} - R_{b,t+1}$	equity premium	0.69	0.07	0.70	0.69
$\ln(C_{t+1}/C_t)$	consumption growth	1.00	0.07	1.01	1.00
Parameter estimates					
$\hat{\beta}$	factor of time preference	1.58	0.15	1.51	1.57
$\hat{\gamma}$	coef. of relative risk aversion	282.37	46.28	263.25	278.44
$\widehat{e_R^b}$	EE error risky bond	0.00	0.00	0.00	0.00
$\widehat{e_X^c}$	EE excess return	0.00	0.00	0.00	0.00
$\widehat{RMSE}$	root mean square error	0.00	0.00	0.00	0.00

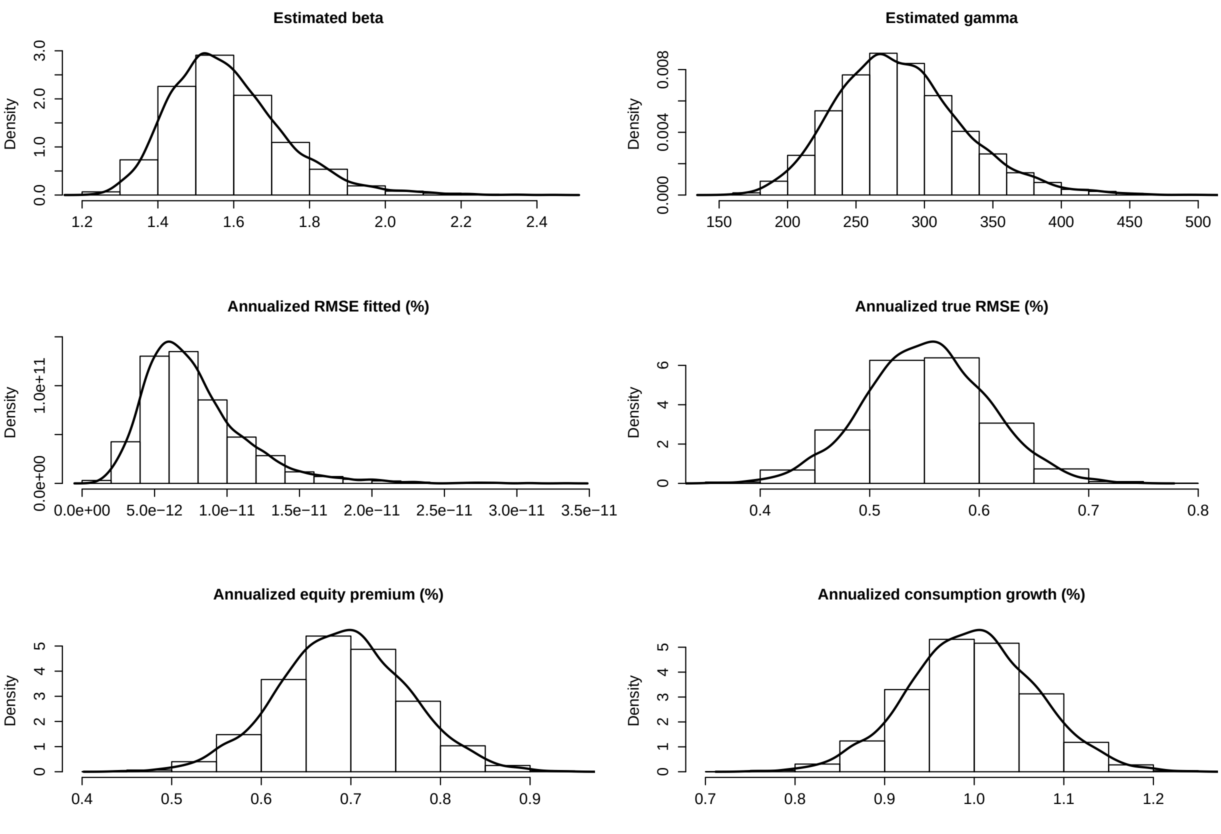


Table 4: C-CAPM simulation results (endowment economy, 37% converged)

Results	analytical solution	conditional (no disasters)			
	parameterization (2)	Mean	Std. dev.	Mode	Median
β	factor of time preference	0.97			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	−6.62	0.28	−6.61	−6.62
e_X^c	EE error excess return	2.73	0.07	2.76	2.73
$RMSE$	root mean square error	5.06	0.20	5.06	5.06
Observed random variables					
$R_{b,t+1}$	bill return	0.25	0.00	0.30	0.25
$R_{c,t+1}$	equity return	3.04	0.07	3.02	3.04
$R_{c,t+1} - R_{b,t+1}$	equity premium	2.79	0.07	2.81	2.79
$\ln(C_{t+1}/C_t)$	consumption growth	0.99	0.07	0.99	0.99
Parameter estimates					
$\hat{\beta}$	factor of time preference	0.19	0.19	0.00	0.12
$\hat{\gamma}$	coef. of relative risk aversion	1425.00	511.77	1037.50	1318.40
$\widehat{e_R^b}$	EE error risky bond	0.00	0.00	0.00	0.00
$\widehat{e_X^c}$	EE excess return	0.00	0.00	0.00	0.00
$\widehat{RMSE}$	root mean square error	0.00	0.00	0.00	0.00

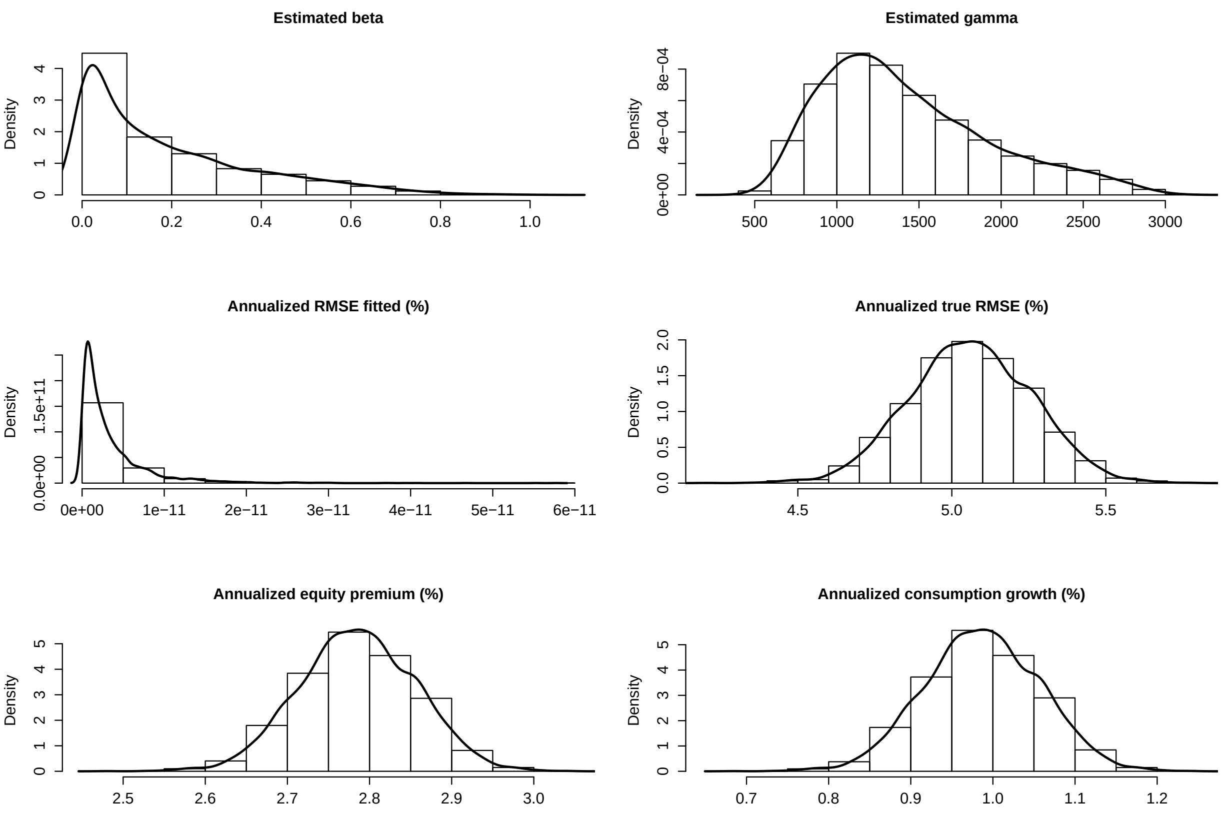


Table 5: C-CAPM simulation results (endowment economy)

Results	analytical solution parameterization (4)	Mean	Std. dev.	Mode	Median
		conditional (no disasters)			
β	factor of time preference	0.97			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	−5.59	0.28	−5.48	−5.59
e_X^c	EE error excess return	1.66	0.07	1.66	1.66
$RMSE$	root mean square error	4.12	0.20	4.04	4.12
Observed random variables					
$R_{b,t+1}$	bill return	1.35	0.00	1.50	1.35
$R_{c,t+1}$	equity return	3.05	0.07	3.05	3.05
$R_{c,t+1} - R_{b,t+1}$	equity premium	1.70	0.07	1.70	1.70
$\ln(C_{t+1}/C_t)$	consumption growth	1.00	0.07	1.00	1.00
Parameter estimates					
$\hat{\beta}$	factor of time preference	1.17	0.17	1.21	1.19
$\hat{\gamma}$	coef. of relative risk aversion	804.64	244.70	752.50	754.20
$\widehat{e_R^b}$	EE error risky bond	0.00	0.00	0.00	0.00
$\widehat{e_X^c}$	EE excess return	0.00	0.00	0.00	0.00
$\widehat{RMSE}$	root mean square error	0.00	0.00	0.00	0.00

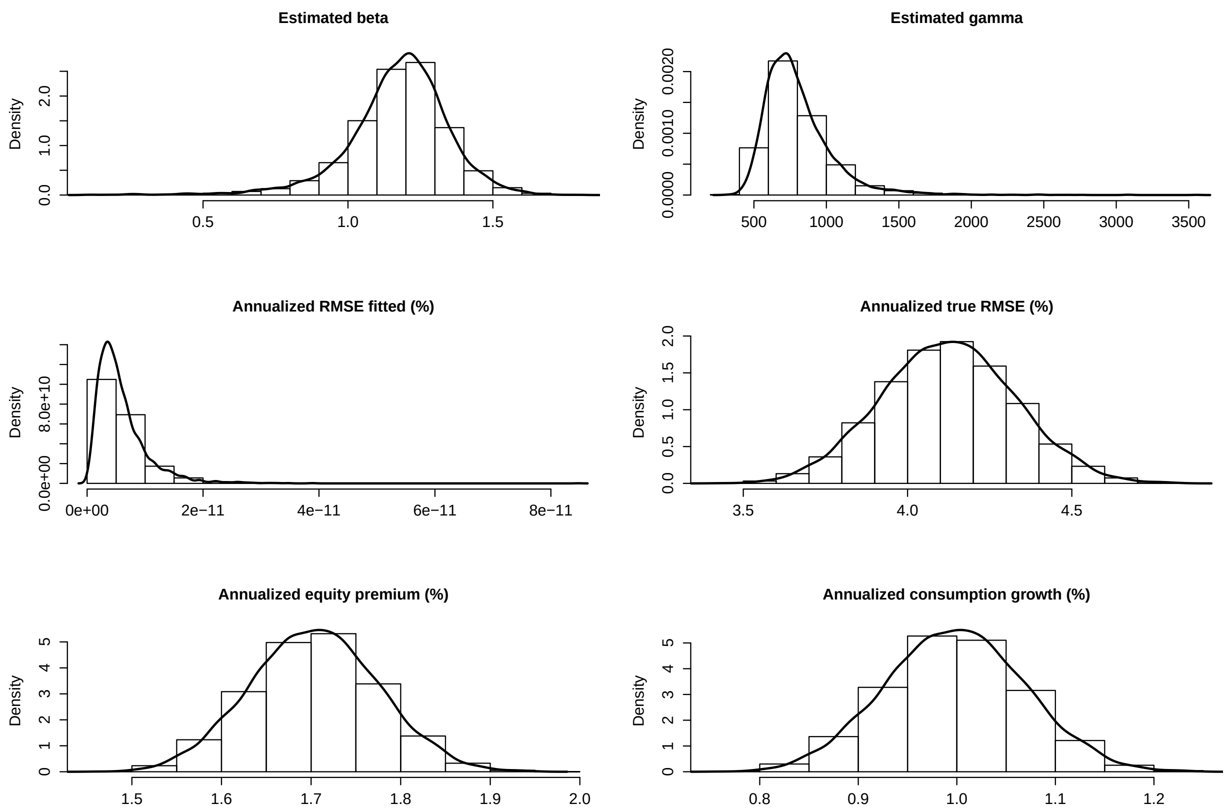


Table 6: C-CAPM simulation results (endowment economy)

Results	analytical solution parameterization (5)	Mean	unconditional Std. dev.	Mode	Median
β	factor of time preference	0.97			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	0.00	0.29	0.06	0.00
e_X^c	EE error excess return	0.00	0.07	−0.02	0.00
$RMSE$	root mean square error	0.17	0.13	0.09	0.14
Observed random variables					
$R_{b,t+1}$	bill return	7.04	0.00	7.50	7.04
$R_{c,t+1}$	equity return	7.05	0.07	7.06	7.05
$R_{c,t+1} - R_{b,t+1}$	equity premium	0.01	0.07	0.01	0.01
$\ln(C_{t+1}/C_t)$	consumption growth	1.00	0.07	0.98	1.00
Parameter estimates					
$\hat{\beta}$	factor of time preference	1.00	0.07	0.98	0.99
$\hat{\gamma}$	coef. of relative risk aversion	3.73	29.37	9.25	3.64
$\widehat{e_R^b}$	EE error risky bond	0.00	0.00	0.00	0.00
$\widehat{e_X^c}$	EE excess return	0.00	0.00	0.00	0.00
$\widehat{RMSE}$	root mean square error	0.00	0.00	0.00	0.00

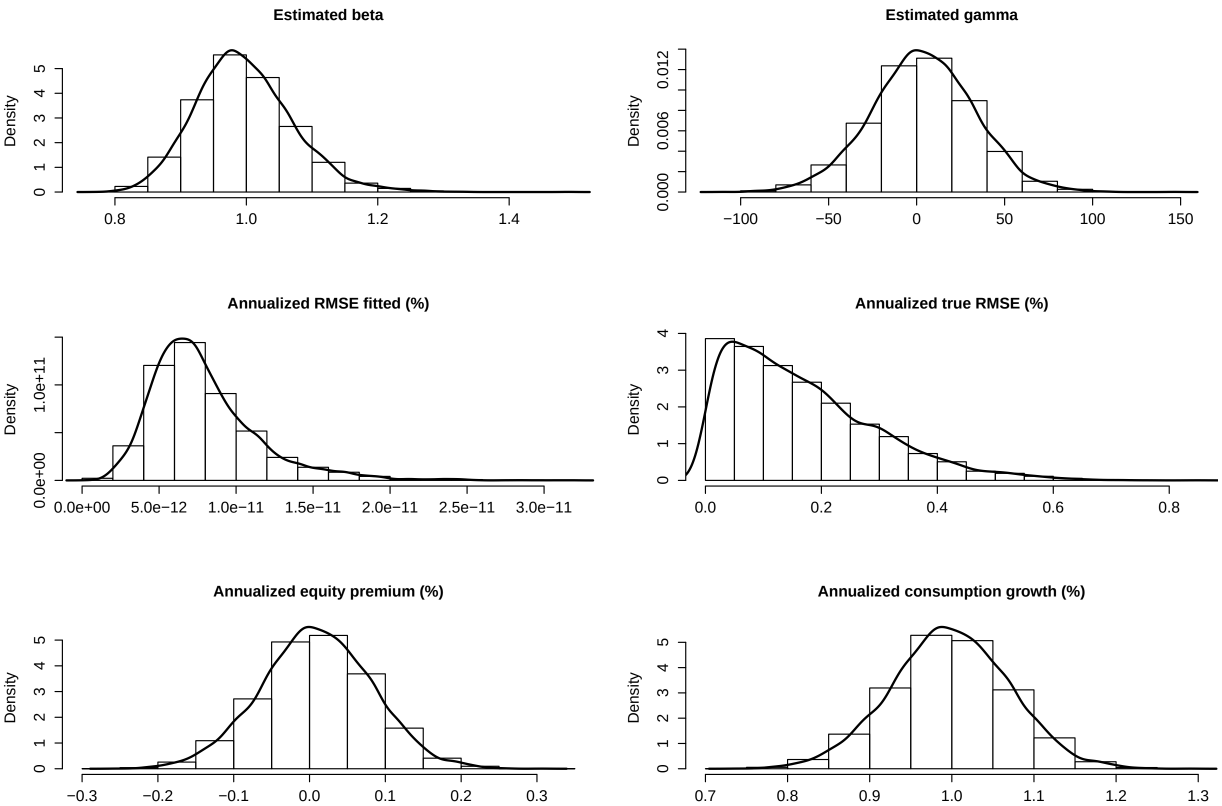


Table 7: C-CAPM simulation results (production economy)

Results	linear-policy-function, parameterization (1)	conditional (no disasters) Mean	Std. dev.	Mode	Median
β	factor of time preference	0.97			
γ	coef. of relative risk aversion	0.50			
e_R^b	EE error risky bond	−0.54	0.04	−0.53	−0.54
e_X^c	EE error excess return	0.53	0.15	0.60	0.53
$RMSE$	root mean square error	0.54	0.08	0.58	0.54
Observed random variables					
$R_{b,t+1}$	bill return (gross)	4.32	0.12	4.34	4.32
$R_{c,t+1}$	equity return (gross)	4.86	0.26	4.86	4.86
$R_{c,t+1} - R_{b,t+1}$	equity premium	0.54	0.15	0.56	0.54
$\ln(C_{t+1}/C_t)$	consumption growth	3.67	0.23	3.71	3.67
Parameter estimates					
$\hat{\beta}$	factor of time preference	35.42	43.13	6.25	18.44
$\hat{\gamma}$	coef. of relative risk aversion	385.45	148.95	312.50	366.28
$\widehat{e_R^b}$	EE error risky bond	0.00	0.00	0.00	0.00
$\widehat{e_X^c}$	EE excess return	0.04	0.11	0.00	0.00
$\widehat{RMSE}$	root mean square error	0.03	0.08	0.00	0.00

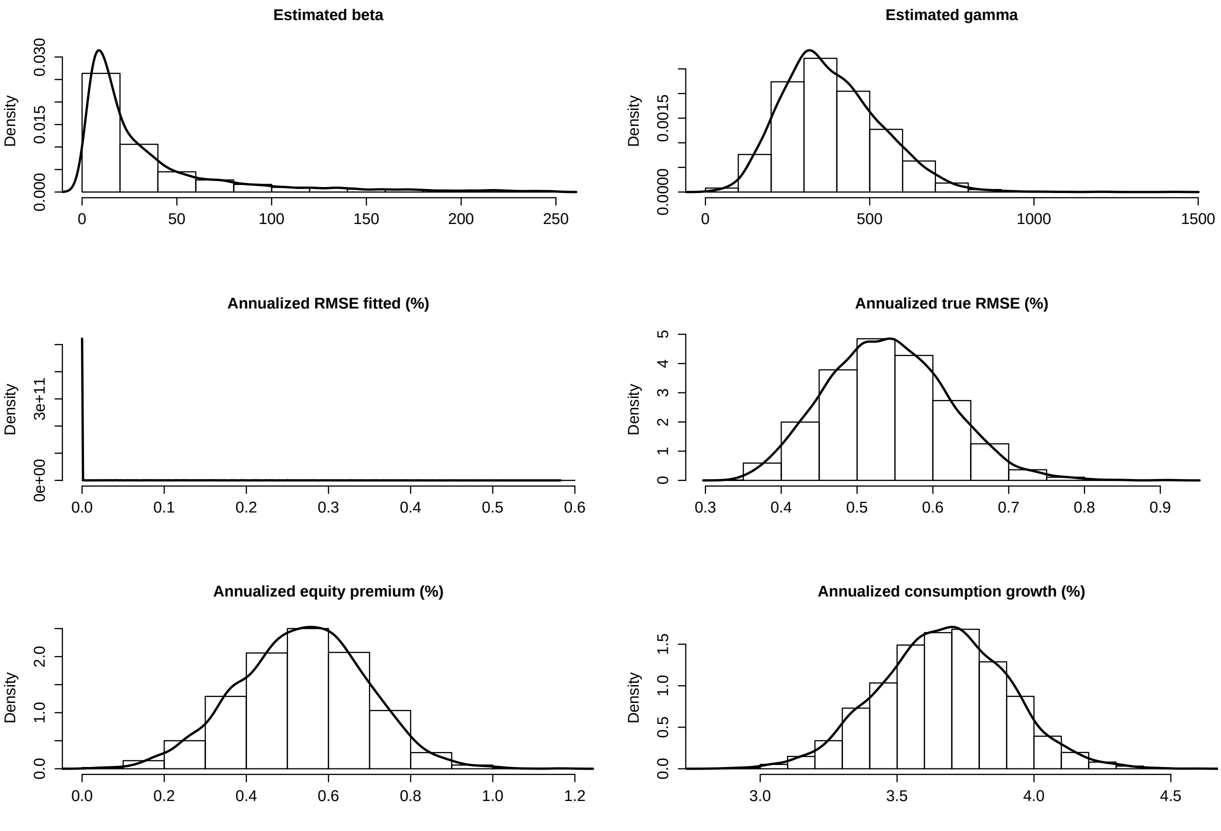


Table 8: C-CAPM simulation results (production economy)

Results	constant-saving-function, parameterization (2)	conditional (no disasters) Mean	Std. dev.	Mode	Median
β	factor of time preference	0.98			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	-4.62	0.65	-4.68	-4.61
e_X^c	EE error excess return	4.64	0.17	4.68	4.64
$RMSE$	root mean square error	4.64	0.35	4.62	4.63
Observed random variables					
$R_{b,t+1}$	bill return (gross)	6.42	0.39	6.43	6.42
$R_{c,t+1}$	equity return (gross)	11.21	0.40	11.35	11.20
$R_{c,t+1} - R_{b,t+1}$	equity premium	4.79	0.17	4.77	4.78
$\ln(C_{t+1}/C_t)$	consumption growth	2.19	0.24	2.09	2.19
Parameter estimates					
$\hat{\beta}$	factor of time preference	0.80	0.64	0.00	0.86
$\hat{\gamma}$	coef. of relative risk aversion	474.36	442.44	175.00	327.02
$\widehat{e_R^b}$	EE error risky bond	-0.03	0.02	0.00	-0.04
$\widehat{e_X^c}$	EE excess return	3.42	1.31	0.00	3.90
$\widehat{RMSE}$	root mean square error	2.42	0.93	0.00	2.76

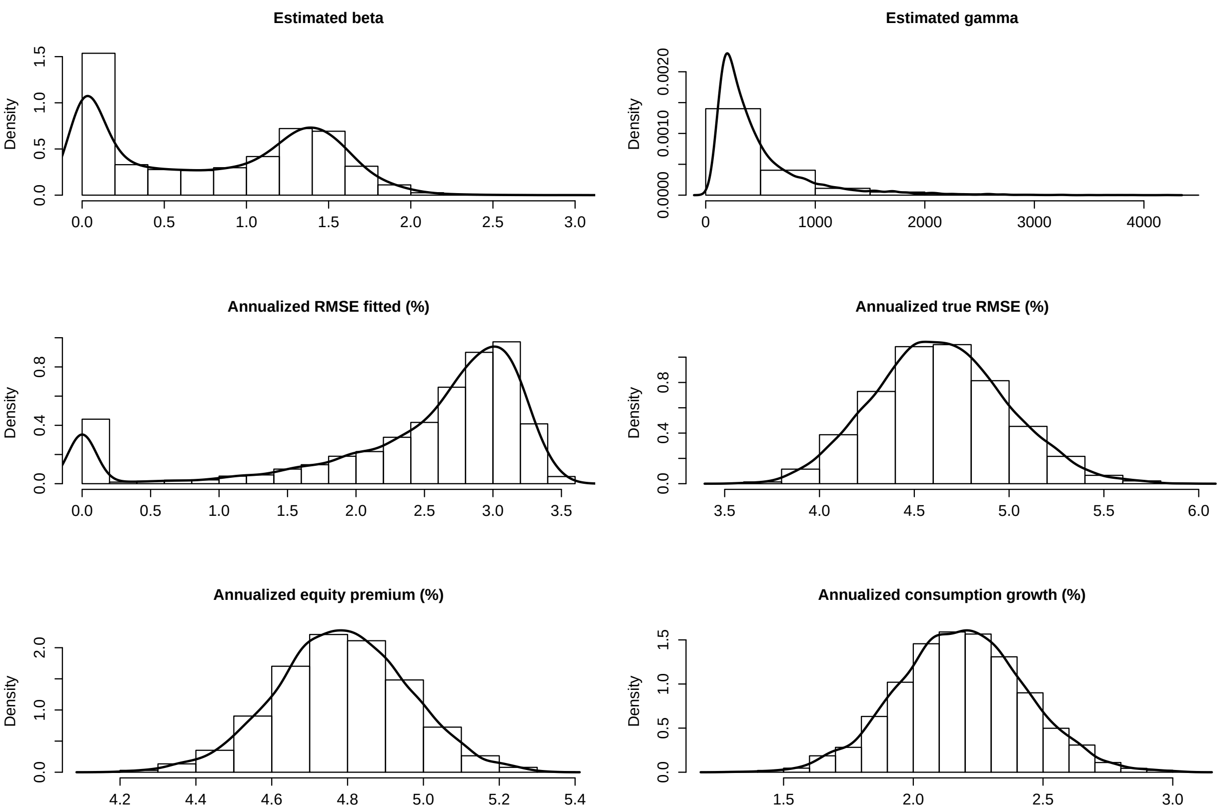


Table 9: C-CAPM simulation results (production economy)

Results	constant-saving-function, parameterization (3)	conditional (no disasters) Mean	Std. dev.	Mode	Median
β	factor of time preference	0.98			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	−2.56	0.64	−2.88	−2.56
e_X^c	EE error excess return	2.68	0.17	2.72	2.68
$RMSE$	root mean square error	2.64	0.34	2.52	2.63
Observed random variables					
$R_{b,t+1}$	bill return (gross)	7.68	0.38	7.61	7.67
$R_{c,t+1}$	equity return (gross)	10.44	0.40	10.46	10.44
$R_{c,t+1} - R_{b,t+1}$	equity premium	2.77	0.17	2.80	2.77
$\ln(C_{t+1}/C_t)$	consumption growth	2.19	0.24	2.23	2.19
Parameter estimates					
$\hat{\beta}$	factor of time preference	0.75	0.61	0.00	0.68
$\hat{\gamma}$	coef. of relative risk aversion	453.07	361.70	195.00	344.39
$\widehat{e_R^b}$	EE error risky bond	−0.01	0.01	0.00	−0.01
$\widehat{e_X^c}$	EE excess return	1.33	1.01	0.00	1.65
$\widehat{RMSE}$	root mean square error	0.94	0.72	0.00	1.17

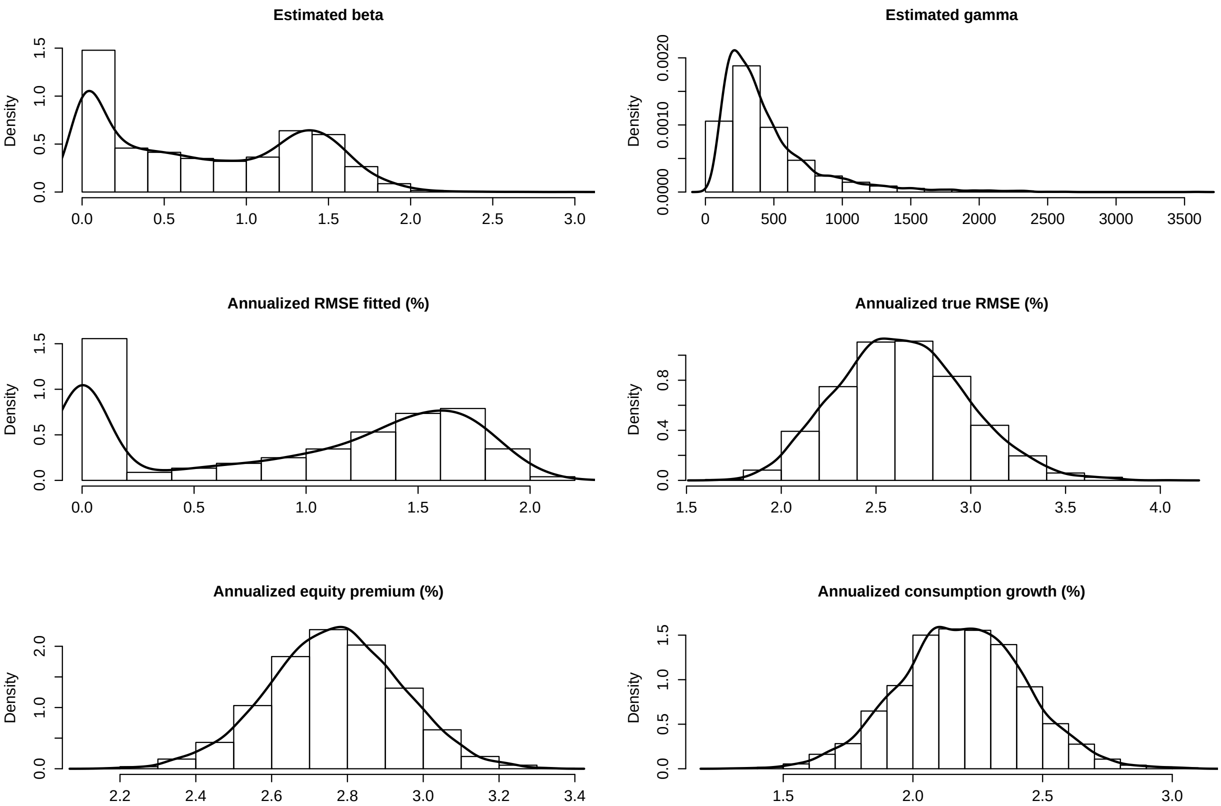


Table 10: C-CAPM simulation results (production economy)

Results	constant-saving-function, parameterization (4)	conditional (no disasters) Mean	Std. dev.	Mode	Median
β	factor of time preference	0.98			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	−4.49	0.58	−4.39	−4.49
e_X^c	EE error excess return	4.63	0.17	4.69	4.63
$RMSE$	root mean square error	4.57	0.32	4.47	4.56
Observed random variables					
$R_{b,t+1}$	bill return (gross)	6.86	0.35	6.81	6.86
$R_{c,t+1}$	equity return (gross)	11.63	0.37	11.88	11.63
$R_{c,t+1} - R_{b,t+1}$	equity premium	4.78	0.18	4.83	4.78
$\ln(C_{t+1}/C_t)$	consumption growth	2.50	0.22	2.55	2.50
Parameter estimates					
$\hat{\beta}$	factor of time preference	0.88	0.87	0.00	0.57
$\hat{\gamma}$	coef. of relative risk aversion	806.11	661.98	325.00	590.05
$\widehat{e_R^b}$	EE error risky bond	−0.03	0.02	0.00	−0.03
$\widehat{e_X^c}$	EE excess return	2.82	1.45	0.00	3.31
$\widehat{RMSE}$	root mean square error	1.99	1.03	0.00	2.34

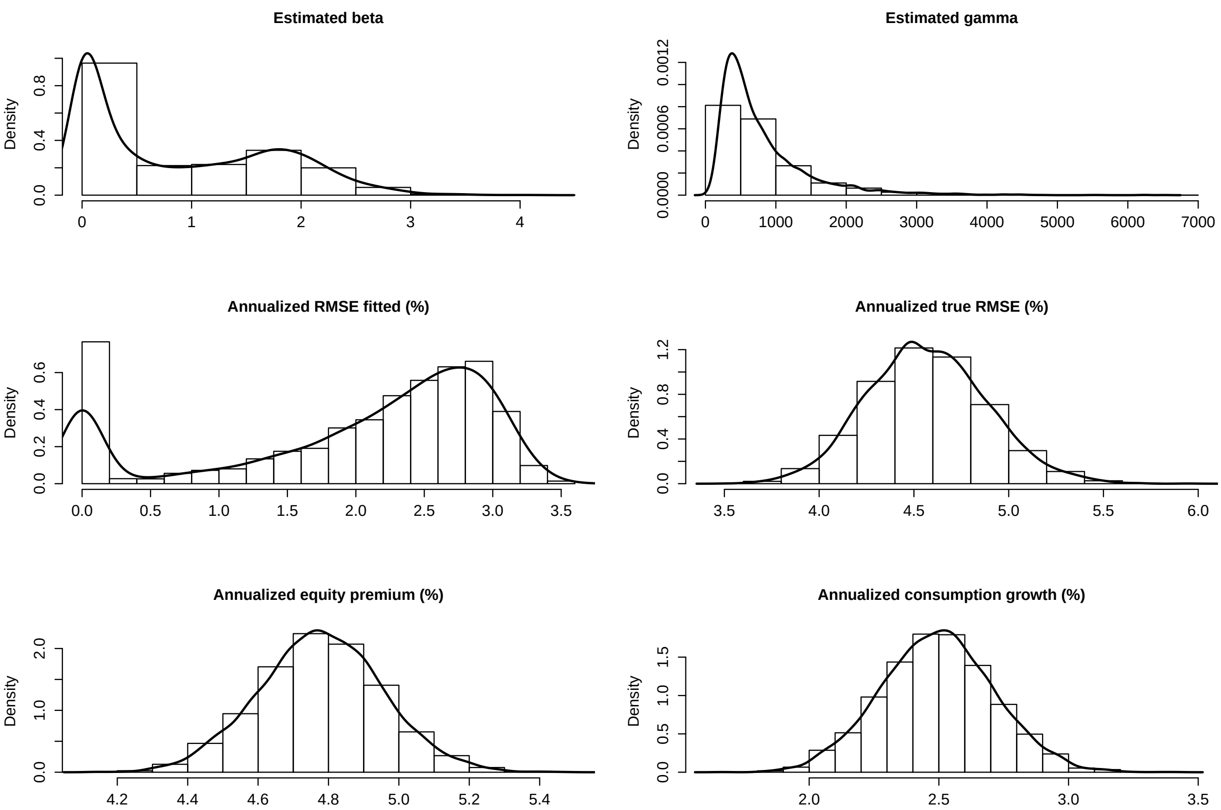
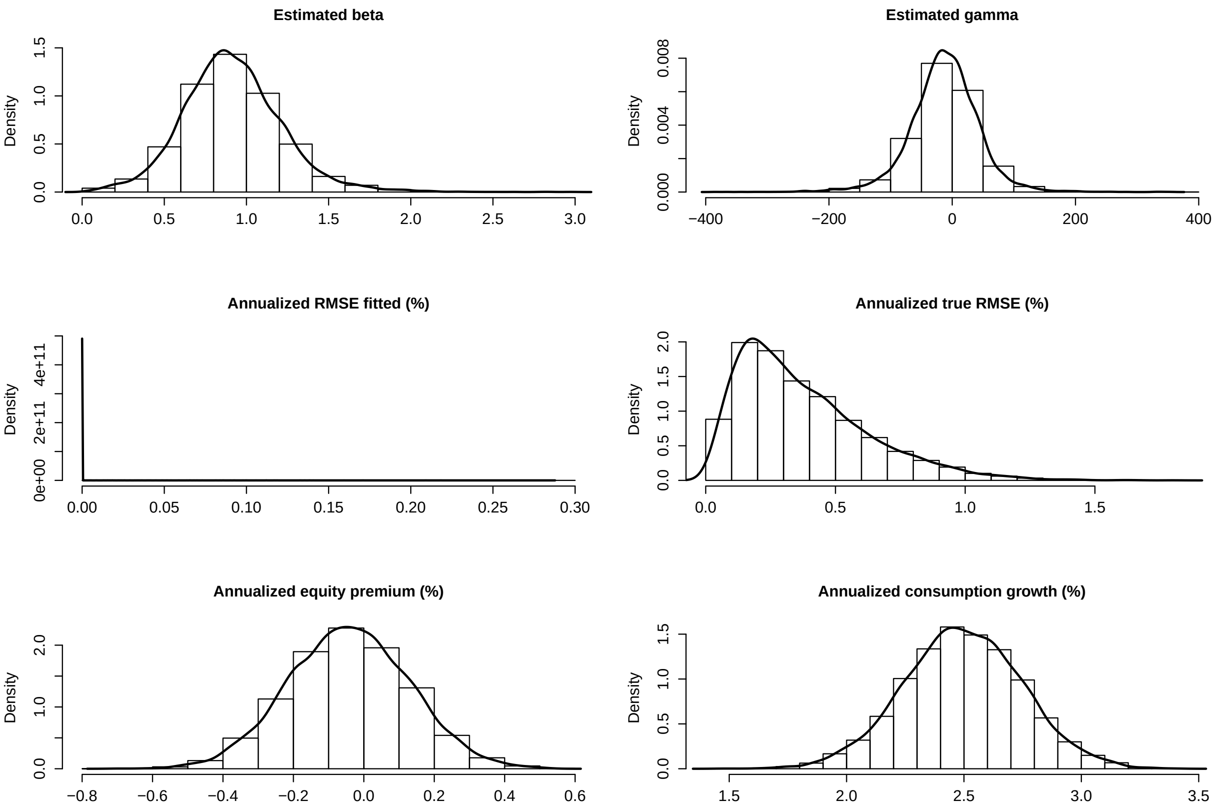


Table 11: C-CAPM simulation results (production economy)

Results	constant-saving-function, parameterization (5)	conditional (no disasters) Mean	Std. dev.	Mode	Median
β	factor of time preference	0.97			
γ	coef. of relative risk aversion	4.00			
e_R^b	EE error risky bond	0.08	0.61	−0.10	0.08
e_X^c	EE error excess return	−0.06	0.17	−0.03	−0.06
$RMSE$	root mean square error	0.37	0.25	0.16	0.32
Observed random variables					
$R_{b,t+1}$	bill return (gross)	13.21	0.48	13.10	13.20
$R_{c,t+1}$	equity return (gross)	13.17	0.48	13.10	13.17
$R_{c,t+1} - R_{b,t+1}$	equity premium	−0.04	0.17	−0.08	−0.04
$\ln(C_{t+1}/C_t)$	consumption growth	2.50	0.25	2.46	2.49
Parameter estimates					
$\hat{\beta}$	factor of time preference	0.92	0.30	0.84	0.90
$\hat{\gamma}$	coef. of relative risk aversion	−12.52	55.94	−17.00	−11.75
$\widehat{e_R^b}$	EE error risky bond	0.00	0.00	0.00	0.00
$\widehat{e_X^c}$	EE excess return	0.00	0.01	0.00	0.00
$\widehat{RMSE}$	root mean square error	0.00	0.01	0.00	0.00



B Appendix

B.1 Computing moments

B.1.1 A lemma for $E\left(c^{kN_s}\right)$

The following lemmas are required to compute the stochastic discount factor in the text.[16]

Lemma B.1 *The conditional mean of c^{kN_s} conditioned on the information set at time t is*

$$E_t\left[c^{kN_s}\right] = c^{kN_t} e^{(c^k-1)\lambda(s-t)}, \quad s > t, \quad c,k = const.$$

Note that for integer k, these are the raw moments of c^{N_s}.

Proof. We can trivially rewrite $c^{kN_s} = c^{kN_t} c^{(N_s-N_t)k}$. Thus, $E_t\left[c^{kN_s}\right] = c^{kN_t} E_t\left[c^{(N_s-N_t)k}\right]$. Computing this expectation requires the probability that a Poisson process jumps n times between t and s. Formally,

$$\begin{aligned} E_t\left[c^{(N_s-N_t)k}\right] &= \sum_{n=0}^{\infty} c^{kn} \frac{e^{-\lambda(s-t)}[(s-t)\lambda]^n}{n!} = \sum_{n=0}^{\infty} \frac{e^{-(s-t)\lambda}[(s-t)c^k\lambda]^n}{n!} \\ &= e^{(s-t)(c^k-1)\lambda} \sum_{n=0}^{\infty} \frac{e^{-(s-t)\lambda-(s-t)(c^k-1)\lambda}[(s-t)c^k\lambda]^n}{n!} \\ &= e^{(s-t)(c^k-1)\lambda} \sum_{n=0}^{\infty} \frac{e^{-(s-t)c^k\lambda}[(s-t)c^k\lambda]^n}{n!} = e^{(s-t)(c^k-1)\lambda}, \end{aligned}$$

where $\frac{e^{-\lambda s}[\lambda s]^n}{n!}$ is the probability of $N_s = n$, and $\sum_{n=0}^{\infty} \frac{e^{-(s-t)c^k\lambda}[(s-t)c^k\lambda]^n}{n!} = 1$ is the probability function over the whole support of the Poisson distribution used in the last step. ■

Lemma B.2 *The unconditional mean of c^{kN_s} is*

$$E\left[c^{(N_s-N_t)k}\right] = e^{(c^k-1)\lambda(s-t)}, \quad s > t, \quad c,k = const.$$

Proof. This proof simply applies lemma B.1. ■

B.2 Lucas' endowment economy with rare disasters

B.2.1 Bellman equation

Choosing the control $C_s \in \mathbb{R}_+$ at time s, the Bellman equation reads

$$\begin{aligned} \rho V(W_s) = \max_{C_s} \big\{ & u(C_s) + (\mu_M W_s - C_s) V_W + \tfrac{1}{2}\sigma_M^2 W_s^2 V_{WW} \\ & + (E^{\zeta}\left[V((1-\zeta_M(s))W_s)\right] - V(W_s))\lambda \big\}. \end{aligned}$$

[16] We are indebted to Ken Sennewald and Klaus Wälde for discussions.

B.2.2 General equilibrium prices

Using the inverse function, we are able to determine the path for consumption ($u'' \neq 0$). From the Euler equation (17), we obtain

$$\begin{aligned} dC_t &= \big((\rho-\mu_M+\lambda)u'(C_t)/u''(C_t) - \sigma_M^2 W_t C_W - \tfrac{1}{2}u'''(C_t)/u''(C_t)\sigma_M^2 W_t^2 C_W^2 \\ &\quad -E^\zeta\left[u'(C((1-\zeta_M(t))W_t))(1-\zeta_M(t))\right]\lambda/u''(C_t)\big)dt \\ &\quad +\sigma_M W_t C_W dB_t + (C((1-\zeta_M(t))W_{t-}) - C(W_{t-}))dN_t, \end{aligned} \tag{59}$$

where we employed the inverse function $c = g(u'(c))$ which has

$$g'(u'(c)) = 1/u''(c), \quad g''(u'(c)) = -u'''(c)/(u''(c))^3.$$

Economically, concave utility ($u'(c) > 0$, $u''(c) < 0$) implies risk aversion, whereas convex marginal utility, $u'''(c) > 0$, implies a positive precautionary saving motive. Accordingly, $-u''(c)/u'(c)$ measures absolute risk aversion, whereas $-u'''(c)/u''(c)$ measures the degree of absolute prudence, i.e., the intensity of the precautionary saving motive (Kimball, 1990).

Because output is perishable, using the market clearing condition $Y_t = C_t = A_t$, and

$$dC_t = \bar{\mu}C_t dt + \bar{\sigma}C_t dB_t + (\exp(\bar{\nu})-1)C_{t-}dN_t, \tag{60}$$

the parameters of price dynamics are pinned down in general equilibrium. In particular, we obtain J_t implicitly as function of $\bar{\nu}$, D_t (stochastic investment opportunities), and the curvature of the consumption function, where $\tilde{C}(W_t) \equiv C((1-\zeta_M(t))W_t)/C(W_t)$ defines optimal consumption jumps. In equilibrium, market clearing requires the percentage jump in aggregate consumption to match the size of the disaster, $\exp(\bar{\nu}) = \tilde{C}(W_t)$, and thus $\exp(\bar{\nu}) = C((1+(J_t-D_t)w_t + D_t)W_t)/C(W_t)$ implies a constant jump size. For example, if consumption is linear homogeneous in wealth, the jump size of the market portfolio is

$$\zeta_M = \zeta_M^0 = e^{\bar{\nu}} - 1. \tag{61}$$

Similarly, the market clearing condition pins down $\sigma_M W_t C_W = \bar{\sigma}C_t$, and

$$\mu_M - r = -\frac{u''(C_t)C_W W_t}{u'(C(W_t))}\sigma_M^2 - \frac{u'(e^{\bar{\nu}}C(W_t))}{u'(C(W_t))}\left((1-e^\kappa)q - \zeta_M\right)\lambda. \tag{62}$$

Inserting our results back into (59), we obtain that consumption follows,

$$\begin{aligned} dC_t &= (\rho - r + \lambda)\frac{u'(C_t)}{u''(C_t)}dt - \tfrac{1}{2}\frac{u'''(C_t)}{u''(C_t)}\sigma_M^2 W_t^2 C_W^2 dt - (1-(1-e^\kappa)q)\frac{u'(e^{\bar{\nu}}C_t)}{u''(C_t)}\lambda dt \\ &\quad +\sigma_M W_t C_W dB_t + (C((1-\zeta_M(t))W_{t-}) - C(W_{t-}))dN_t. \end{aligned}$$

This in turn determines the return on the government bill

$$r = \rho - \frac{u''(C_t)C_t}{u'(C_t)}\bar{\mu} - \tfrac{1}{2}\frac{u'''(C_t)C_t^2}{u'(C_t)}\bar{\sigma}^2 + \lambda - (1-(1-e^\kappa)q)\frac{u'(e^{\bar{\nu}}C_t)}{u'(C_t)}\lambda. \tag{63}$$

B.2.3 General equilibrium consumption growth rates and asset returns

Consumption. Consumption growth rates are exogenous in the endowment economy. Thus, consumption growth rates can be obtained from the dividend process (7),

$$\begin{aligned} A_s &= A_t e^{(\bar{\mu}-\frac{1}{2}\bar{\sigma}^2)(s-t)+\bar{\sigma}(B_s-B_t)+\bar{\nu}(N_s-N_t)} & (64) \\ \Leftrightarrow \ln(C_s/C_t) &= \ln(A_s/A_t) = (\bar{\mu}-\tfrac{1}{2}\bar{\sigma}^2)(s-t)+\bar{\sigma}(B_s-B_t)+\bar{\nu}(N_s-N_t). & (65) \end{aligned}$$

Risky asset. Consider a claim which pays a dividend $X_{c,t+1} = A_{t+1}$, i.e., an instantaneous return in period $s = t+1$,

$$R_{c,t+1} = \frac{A_{t+1}}{P_{c,t}}. \tag{66}$$

From (2) we obtain the price of this asset in terms of the consumption good as

$$\begin{aligned} P_{c,t} &= E_t\left[\frac{m_{t+1}}{m_t}A_{t+1}\right] \\ &= e^{-(\rho+(\gamma-1)\bar{\mu}+\frac{1}{2}(1-\gamma)\bar{\sigma}^2)}E_t\left[e^{(1-\gamma)\bar{\sigma}(B_{t+1}-B_t)}\right]E_t\left[e^{(1-\gamma)\bar{\nu}(N_{t+1}-N_t)}\right]A_t \\ &= e^{-\rho+(1-\gamma)\bar{\mu}-\frac{1}{2}(1-\gamma)\bar{\sigma}^2+\frac{1}{2}(1-\gamma)^2\bar{\sigma}^2+(e^{(1-\gamma)\bar{\nu}}-1)\lambda}A_t. \end{aligned}$$

Inserting this result together with (64) into (66) finally gives (27).

Riskless asset. Consider a riskless asset which is subject to default risk. Use the random payoff $X_{b,t+1} = A_t e^{r+\int_t^{t+1}\ln(1+D_s)dN_s}$ and (2) which gives the price of the government bill as

$$\begin{aligned} P_{b,t} &= E_t\left[\frac{m_{t+1}}{m_t}A_t e^{r+\int_t^{t+1}\ln(1+D_s)dN_s}\right] \\ &= e^{-(\rho-r+\gamma\bar{\mu}-\frac{1}{2}\gamma\bar{\sigma}^2)}E_t\left[e^{-\gamma\bar{\sigma}(B_{t+1}-B_t)}\right]E_t\left[e^{(e^{\ln(1+D_t)-\gamma\bar{\nu}}-1)\lambda}\right]A_t \\ &= e^{-(\rho-r+\gamma\bar{\mu}-\frac{1}{2}\gamma\bar{\sigma}^2)+\frac{1}{2}(\gamma\bar{\sigma})^2+q(e^{\kappa-\gamma\bar{\nu}}-1)\lambda+(1-q)(e^{-\gamma\bar{\nu}}-1)\lambda}A_t = A_t. \end{aligned}$$

This in turn gives the return of the government bill with default risk.

B.3 A production economy with rare events

B.3.1 The Bellman equation and the Euler equation

As a necessary condition for optimality the Bellman's principle gives at time s

$$\rho V(W_s, A_s) = \max_{C_s}\left\{u(C_s) + \frac{1}{dt}E_s dV(W_s, A_s)\right\}.$$

Using Itô's formula yields

$$\begin{aligned} dV(W_s,A_s) &= V_W(dW_s - J_sW_{s-}dN_t) + V_A(dA_s - ((\exp(\bar{\nu})-1)A_{t-}d\bar{N}_t)) \\ &\quad + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2A_s^2 + V_{WW}\sigma^2W_s^2\right)dt \\ &\quad + [V(W_s,A_s) - V(W_{s-},A_{s-})](d\bar{N}_t + dN_t) \\ &= ((r_s-\delta)W_s + w_s^L - C_s)V_W dt + V_W\sigma W_s dZ_s + V_A\bar{\mu}A_s dt + V_A\bar{\sigma}A_s dB_s \\ &\quad + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2A_s^2 + V_{WW}\sigma^2W_s^2\right)dt + [V(e^{\nu}W_{s-},A_{s-}) - V(W_{s-},A_{s-})]dN_t \\ &\quad + [V(W_{s-},e^{\bar{\nu}}A_{s-}) - V(W_{s-},A_{s-})]d\bar{N}_t. \end{aligned}$$

Using the property of stochastic integrals, we may write

$$\begin{aligned} \rho V(W_s,A_s) = \max_{C_s}\Big\{ & u(c_s) + ((r_s-\delta)W_s + w_s^L - C_s)V_W + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2A_s^2 + V_{WW}\sigma^2W_s^2\right) \\ & + V_A\bar{\mu}A_s + [V(e^{\nu}W_s,A_s) - V(W_s,A_s)]\lambda + [V(W_s,e^{\bar{\nu}}A_s) - V(W_s,A_s)]\bar{\lambda}\Big\} \end{aligned}$$

for any $s \in [0,\infty)$. Because it is a necessary condition for optimality, we obtain the first-order condition (35) which makes optimal consumption a function of the state variables.

For the *evolution of the costate* we use the maximized Bellman equation

$$\begin{aligned} \rho V(W_t,A_t) &= u(C(W_t,A_t)) + ((r_t-\delta)W_t + w_t^L - C(W_t,A_t))V_W + V_A\bar{\mu}A_t \\ &\quad + \tfrac{1}{2}\left(V_{AA}\bar{\sigma}^2A_t^2 + V_{WW}\sigma^2W_t^2\right) + [V(e^{\nu}W_t,A_t) - V(W_t,A_t)]\lambda \\ &\quad + [V(W_t,e^{\bar{\nu}}A_t) - V(W_t,A_t)]\bar{\lambda}, \end{aligned} \tag{67}$$

where $r_t = r(W_t,A_t)$ and $w_t^L = w(W_t,A_t)$ follow from the firm's optimization problem, and the envelope theorem (also for the factor rewards) to compute the costate,

$$\begin{aligned} \rho V_W &= \bar{\mu}A_tV_{AW} + ((r_t-\delta)W_t + w_t^L - C_t)V_{WW} + (r_t-\delta)V_W + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2A_t^2 + V_{WWW}\sigma^2W_t^2\right) \\ &\quad + V_{WW}\sigma^2W_t + [V_W(e^{\nu}W_t,A_t)e^{\nu} - V_W(W_t,A_t)]\lambda + [V_W(W_t,e^{\bar{\nu}}A_t) - V_W(W_t,A_t)]\bar{\lambda}. \end{aligned}$$

Collecting terms we obtain

$$\begin{aligned} (\rho - (r_t-\delta) + \lambda + \bar{\lambda})V_W &= V_{AW}\bar{\mu}A_t + ((r_t-\delta)W_t + w_t^L - C_t)V_{WW} \\ &\quad + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2A_t^2 + V_{WWW}\sigma^2W_t^2\right) \\ &\quad + \sigma^2V_{WW}W_t + V_W(e^{\nu}W_t,A_t)e^{\nu}\lambda + V_W(W_t,e^{\bar{\nu}}A_t)\bar{\lambda}. \end{aligned}$$

Using Itô's formula, the costate obeys

$$\begin{aligned} dV_W &= V_{AW}\bar{\mu}A_t dt + V_{AW}\bar{\sigma}A_t dB_t + \tfrac{1}{2}\left(V_{WAA}\bar{\sigma}^2A_t^2 + V_{WWW}\sigma^2W_t^2\right)dt + V_{WW}\sigma W_t dZ_t \\ &\quad + ((r_t-\delta)W_t + w_t^L - C_t)V_{WW}dt + [V_W(W_t,A_t) - V_W(W_{t-},A_{t-})](d\bar{N}_t + dN_t) \end{aligned}$$

where inserting yields

$$
\begin{aligned}
dV_W &= (\rho - (r_t - \delta) + \lambda + \bar{\lambda})V_W dt - V_W(e^{\nu}W_t, A_t)e^{\nu}\lambda - V_W(W_t, e^{\bar{\nu}}A_t)\bar{\lambda} \\
&\quad -\sigma^2 V_{WW}W_t dt + V_{AW}A_t\bar{\sigma}dB_t + V_{WW}W_t\sigma dZ_t \\
&\quad +[V_W(e^{\nu}W_{t-}, A_{t-}) - V_W(W_{t-}, A_{t-})]dN_t + [V_W(W_{t-}, e^{\bar{\nu}}A_{t-}) - V_W(W_{t-}, A_{t-})]d\bar{N}_t,
\end{aligned}
$$

which describes the evolution of the costate variable. As a final step, we insert the first-order condition (35) to obtain the Euler equation (36).

B.3.2 Proof of Proposition 3.4

The idea of this proof is to show that using an educated guess of the value function, the maximized Bellman equation (67) and the first-order condition (35) are both fulfilled. We guess that the value function reads

$$
V(W_t, A_t) = \frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta} + f(A_t). \tag{68}
$$

From (35), optimal consumption is a constant fraction of wealth,

$$
C_t^{-\theta} = \mathbb{C}_1 W_t^{-\theta} \quad \Leftrightarrow \quad C_t = \mathbb{C}_1^{-1/\theta} W_t.
$$

Now use the maximized Bellman equation (67), the property of the Cobb-Douglas technology, $F_K = \alpha A_t K_t^{\alpha-1} L^{1-\alpha}$ and $F_L = (1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha}$, together with the transformation $K_t \equiv LW_t$, and insert the solution candidate,

$$
\begin{aligned}
\rho\frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta} &= \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{1-\theta}}{1-\theta} + (\alpha A_t W_t^{\alpha-1}W_t - \delta W_t + (1-\alpha)A_t W_t^{\alpha} - \mathbb{C}_1^{-1/\theta}W_t)\mathbb{C}_1 W_t^{-\theta} \\
&\quad -\tfrac{1}{2}\theta\mathbb{C}_1 W_t^{1-\theta}\sigma^2 - g(A_t) + (e^{(1-\theta)\nu} - 1)\frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta}\lambda,
\end{aligned}
$$

where we defined $g(A_t) \equiv \rho f(A_t) - f_A\bar{\mu}A_t - \frac{1}{2}f_{AA}\bar{\sigma}^2 A_t^2 - [f(e^{\bar{\nu}}A_t) - f(A_t)]\bar{\lambda}$. When imposing the condition $\alpha = \theta$ and $g(A_t) = \mathbb{C}_1 A_t$ it can be simplified to

$$
\begin{aligned}
(\rho - (e^{(1-\theta)\nu} - 1)\lambda)\frac{\mathbb{C}_1 W_t^{1-\theta}}{1-\theta} + g(A_t) &= \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{1-\theta}}{1-\theta} + (A_t W_t^{\alpha-\theta} - \delta W_t^{1-\theta} - \mathbb{C}_1^{-1/\theta}W_t^{1-\theta})\mathbb{C}_1 \\
&\quad -\tfrac{1}{2}\theta\mathbb{C}_1 W_t^{1-\theta}\sigma^2 \\
\Leftrightarrow\ (\rho - (e^{(1-\theta)\nu} - 1)\lambda)W_t^{1-\theta} &= \theta\mathbb{C}_1^{-1/\theta}W_t^{1-\theta} - (1-\theta)\delta W_t^{1-\theta} - \tfrac{1}{2}\theta(1-\theta)W_t^{1-\theta}\sigma^2,
\end{aligned}
$$

which implies that $\mathbb{C}_1^{-1/\theta} = \left(\rho - (e^{(1-\theta)\nu} - 1)\lambda + (1-\theta)\delta + \frac{1}{2}\theta(1-\theta)\sigma^2\right)/\theta$. This proves that the guess (68) indeed is a solution, and by inserting the guess together with the constant, we obtain the optimal policy function for consumption.

B.3.3 Proof of Proposition 3.6

The idea of this proof follows Section B.3.2. An educated guess of the value function is

$$V(W_t, A_t) = \frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1-\alpha\theta} A_t^{-\theta}. \tag{69}$$

From (35), optimal consumption is a constant fraction of income, $C_t^{-\theta} = \mathbb{C}_1 W_t^{-\alpha\theta} A_t^{-\theta}$ or equivalently $C_t = \mathbb{C}_1^{-1/\theta} W_t^{\alpha} A_t$. Now use the maximized Bellman equation (67), the property of the Cobb-Douglas technology, $F_K = \alpha A_t K_t^{\alpha-1} L^{1-\alpha}$ and $F_L = (1-\alpha) A_t K_t^{\alpha} L^{-\alpha}$, together with the transformation $K_t \equiv L W_t$, and insert the solution candidate,

$$\begin{aligned}\rho V(W_t, A_t) \quad = \quad & \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{\alpha-\alpha\theta} A_t^{1-\theta}}{1-\theta} + ((r_t - \delta) W_t + w_t^L - C(W_t, A_t)) V_W + V_A \bar{\mu} A_t \\ & + \tfrac{1}{2}\left(V_{AA} \bar{\sigma}^2 A_t^2 + V_{WW} \sigma^2 W_t^2\right) + [V(e^{\nu} W_t, A_t) - V(W_t, A_t)]\lambda \\ & + [V(W_t, e^{\bar{\nu}} A_t) - V(W_t, A_t)]\bar{\lambda}.\end{aligned}$$

Inserting the guess and collecting terms which is equivalent to

$$\begin{aligned}(\rho - (e^{(1-\alpha\theta)\nu} - 1)\lambda - (e^{-\theta\bar{\nu}} - 1)\bar{\lambda}) \frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1-\alpha\theta} A_t^{-\theta} = & \\ \frac{\mathbb{C}_1^{-\frac{1-\theta}{\theta}} W_t^{\alpha-\alpha\theta} A_t^{1-\theta}}{1-\theta} - \theta \frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1-\alpha\theta} \bar{\mu} A_t^{-\theta} & \\ + \left(\alpha A_t W_t^{\alpha} - \delta W_t + (1-\alpha) A_t W_t^{\alpha} - \mathbb{C}_1^{-1/\theta} W_t^{\alpha} A_t\right) \mathbb{C}_1 W_t^{-\alpha\theta} A_t^{-\theta} & \\ + \tfrac{1}{2}\left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\theta)\sigma^2\right) \frac{\mathbb{C}_1 W_t^{1-\alpha\theta}}{1-\alpha\theta} A_t^{-\theta}. &\end{aligned}$$

Collecting terms gives

$$\begin{aligned}\rho + \theta\bar{\mu} - \tfrac{1}{2}\left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\theta)\sigma^2\right) + (1-\alpha\theta)\delta - (e^{(1-\alpha\theta)\nu} - 1)\lambda - (e^{-\theta\bar{\nu}} - 1)\bar{\lambda} = & \\ \left(\frac{\theta}{1-\theta} \mathbb{C}_1^{-1/\theta} + 1\right)(1-\alpha\theta) A_t W_t^{\alpha-1}, &\end{aligned}$$

which has a solution for $\mathbb{C}_1^{-1/\theta} = (\theta - 1)/\theta$ and

$$\rho = (e^{-\theta\bar{\nu}} - 1)\bar{\lambda} + (e^{(1-\alpha\theta)\nu} - 1)\lambda - \theta\bar{\mu} + \tfrac{1}{2}\left(\theta(1+\theta)\bar{\sigma}^2 - \alpha\theta(1-\alpha\theta)\sigma^2\right) - (1-\alpha\theta)\delta.$$

This proves that the guess (69) indeed is a solution, and by inserting the guess together with the constant, we obtain the optimal policy function for consumption.

B.3.4 Obtaining the rental rate of capital

The rental rate of capital, $r_t = Y_K$, in a neoclassical Cobb-Douglas economy where output is defined as $Y_t \equiv A_t Y(K_t, L) = A_t K_t^{\alpha} L^{1-\alpha}$ follows from the stochastic differential

$$\begin{aligned} dA_t K_t^{\alpha-1} &= (\alpha-1)A_t K_t^{\alpha-2}(Y_t - C_t - \delta K_t)dt + (\alpha-1)\sigma A_t K_t K_t^{\alpha-2} dZ_t \\ &\quad + (A_t K_t^{\alpha-1} - A_{t-}K_{t-}^{\alpha-1})(dN_t + d\bar{N}_t) + \tfrac{1}{2}(\alpha-1)(\alpha-2)K_t^{\alpha-3}\sigma^2 K_t^2 A_t dt \\ &\quad + K_t^{\alpha-1}(dA_t - (\exp(\bar{\nu})-1)A_{t-}d\bar{N}_t) \\ &= (\alpha-1)(Y_t/K_t - C_t/K_t - \delta)A_t K_t^{\alpha-1}dt + (\alpha-1)\sigma A_t K_t^{\alpha-1} dZ_t \\ &\quad + (\exp((\alpha-1)\nu)-1)A_{t-}K_{t-}^{\alpha-1}dN_t + \tfrac{1}{2}(\alpha-1)(\alpha-2)A_t K_t^{\alpha-1}\sigma^2 dt \\ &\quad + \bar{\mu}A_t K_t^{\alpha-1}dt + \bar{\sigma}A_t K_t^{\alpha-1}dB_t + (\exp(\bar{\nu})-1)A_{t-}K_{t-}^{\alpha-1}d\bar{N}_t. \end{aligned}$$

It implies

$$\begin{aligned} dr_t &= \tfrac{1-\alpha}{\alpha}\big(\alpha C_t/K_t + \alpha\delta - \tfrac{1}{2}\alpha(\alpha-2)\sigma^2 - \tfrac{\alpha}{\alpha-1}\bar{\mu} - r_t\big)r_t dt + (\alpha-1)\sigma r_t dZ_t + \bar{\sigma} r_t dB_t \\ &\quad + (\exp((\alpha-1)\nu)-1)r_{t-}dN_t + (\exp(\bar{\nu})-1)r_{t-}d\bar{N}_t, \end{aligned}$$

which for our explicit solutions is a reducible stochastic differential equation.

For $\alpha = \gamma$ we obtain

$$\begin{aligned} dr_t &= \tfrac{1-\alpha}{\alpha}\big(\alpha\phi + \alpha\delta - \tfrac{1}{2}\alpha(\alpha-2)\sigma^2 - \tfrac{\alpha}{\alpha-1}\bar{\mu} - r_t\big)r_t dt + (\alpha-1)\sigma r_t dZ_t + \bar{\sigma} r_t dB_t \\ &\quad + (\exp((\alpha-1)\nu)-1)r_{t-}dN_t + (\exp(\bar{\nu})-1)r_{t-}d\bar{N}_t, \\ &\equiv c_1\big(c_2 - r_t\big)r_t dt + (\alpha-1)\sigma r_t dZ_t + \bar{\sigma} r_t dB_t + (\exp((\alpha-1)\nu)-1)r_{t-}dN_t \\ &\quad + (\exp(\bar{\nu})-1)r_{t-}d\bar{N}_t, \end{aligned}$$

where we defined $c_1 \equiv \frac{1-\alpha}{\alpha}$ and $c_2 \equiv \alpha\phi + \alpha\delta - \frac{1}{2}\alpha(\alpha-2)\sigma^2 - \frac{\alpha}{\alpha-1}\bar{\mu}$.

For $\rho = \bar{\rho}$ we obtain

$$\begin{aligned} dr_t &= \tfrac{1-\alpha}{\alpha}\big(\alpha\delta - \tfrac{1}{2}\alpha(\alpha-2)\sigma^2 - \tfrac{\alpha}{\alpha-1}\bar{\mu} - s r_t\big)r_t dt + (\alpha-1)\sigma r_t dZ_t + \bar{\sigma} r_t dB_t \\ &\quad + (\exp((\alpha-1)\nu)-1)r_{t-}dN_t + (\exp(\bar{\nu})-1)r_{t-}d\bar{N}_t \\ &\equiv c_1\big(c_2 - r_t\big)r_t dt + (\alpha-1)\sigma r_t dZ_t + \bar{\sigma} r_t dB_t + (\exp((\alpha-1)\nu)-1)r_{t-}dN_t \\ &\quad + (\exp(\bar{\nu})-1)r_{t-}d\bar{N}_t, \end{aligned}$$

where we defined $c_1 \equiv \frac{1-\alpha}{\alpha\gamma}$ and $c_2 \equiv \alpha\gamma\delta - \frac{1}{2}\alpha\gamma(\alpha-2)\sigma^2 - \frac{\alpha\gamma}{\alpha-1}\bar{\mu}$.

Because the stochastic differential equation for r_t is reducible, it has the solution

$$r_s = \Theta_{s,t}\left(r_t^{-1} + c_1\int_t^s \Theta_{v,t}dv\right)^{-1},$$

where $\Theta_{s,t} \equiv e^{(c_1c_2 - \frac{1}{2}((\alpha-1)\sigma)^2 - \frac{1}{2}\bar{\sigma}^2)(s-t) + (Z_s - Z_t)(\alpha-1)\sigma + (B_s - B_t)\bar{\sigma} + (\alpha-1)\nu(N_s - N_t) + \bar{\nu}(\bar{N}_s - \bar{N}_t)}$. Observe that the closed-form solution simplifies the problem of simulating Euler equation errors.

B.3.5 General equilibrium consumption growth rates and asset returns

Consumption. Observe that the solution to (29) is for $s \geq t$

$$\begin{aligned} A_s &= A_t e^{(\bar{\mu}-\frac{1}{2}\bar{\sigma}^2)(s-t)+\bar{\sigma}(B_s-B_t)+\bar{\nu}(\bar{N}_s-\bar{N}_t)} \\ \Leftrightarrow \ln(A_s/A_t) &= (\bar{\mu}-\tfrac{1}{2}\bar{\sigma}^2)(s-t)+\bar{\sigma}(B_s-B_t)+\bar{\nu}(\bar{N}_s-\bar{N}_t). \end{aligned} \tag{70}$$

Similarly, we obtain growth rates of the capital stock from (30)

$$\ln(K_t/K_s) = \int_s^t \left(r_v/\alpha - C_v/K_v - \delta - \tfrac{1}{2}\sigma^2\right)dv + \sigma(Z_t - Z_s) + \nu(N_t - N_s). \tag{71}$$

For the case of $\alpha = \gamma$, as from Proposition 3.4, consumption is a linear function in the capital stock $C_t = \phi K_t$. Hence, the consumption growth rate is $\ln(C_s/C_t)|_{\alpha=\gamma} = \ln(K_t/K_s)$ which gives (43). For the case of $\rho = \bar{\rho}$, as from Proposition 3.6, consumption is a constant fraction of output, $C_t = (1-s)Y_t$, and thus we obtain the consumption growth rate as $\ln(C_s/C_t)|_{\rho=\bar{\rho}} = \ln(Y_s/Y_t) = \ln(A_s/A_t) + \alpha\ln(K_s/K_t)$, which finally gives (44).

Risky assets. Consider a risky bond that pays continuously at the rate, r_t. Investing into this asset gives the random payoff $X_{b,t+1} = e^{\int_t^{t+1} r_s ds}$. From (2) we obtain the price

$$\begin{aligned} P_{b,t} &= E_t\left[\frac{m_{t+1}}{m_t} e^{\int_t^{t+1} r_s ds}\right] \\ \Rightarrow \quad P_{b,t}|_{\alpha=\gamma} &= e^{\delta+\gamma\sigma^2+e^{-\gamma\nu}\lambda-e^{(1-\gamma)\nu}\lambda}, \quad P_{b,t}|_{\rho=\bar{\rho}} = e^{\delta+\gamma\alpha\sigma^2+e^{-\alpha\gamma\nu}\lambda-e^{(1-\alpha\gamma)\nu}\lambda}. \end{aligned}$$

Hence, we obtain the returns for the bond as in (46) and (47).

Consider a claim on *output* which pays $X_{c,t+1} = A_{t+1}K_{t+1}^{\alpha}$, i.e., an instantaneous return in period $s = t+1$,

$$R_{c,t+1} = \frac{A_{t+1}K_{t+1}^{\alpha}}{P_{c,t}}. \tag{72}$$

where from (70) and (71), we obtain output at date $s \geq t$ from

$$A_s K_s^{\alpha} = A_t K_t^{\alpha} e^{(\bar{\mu}-\frac{1}{2}\bar{\sigma}^2)(s-t)+\int_t^s (r_v-\alpha C_v/K_v-\alpha\delta-\alpha\frac{1}{2}\sigma^2)dv+\bar{\sigma}(B_s-B_t)+\alpha\sigma(Z_s-Z_t)+\alpha\nu(N_s-N_t)+\bar{\nu}(\bar{N}_s-\bar{N}_t)}.$$

From (2) we obtain the price of this asset in terms of the consumption good as

$$\begin{aligned} P_{c,t} &= E_t\left[\frac{m_{t+1}}{m_t} A_{t+1}K_{t+1}^{\alpha}\right] \\ \Rightarrow \quad P_{c,t}|_{\alpha=\gamma} &= A_t K_t^{\alpha} E_t\left[e^{\bar{\mu}-\frac{1}{2}\bar{\sigma}^2-\alpha\phi-\alpha\delta-\alpha\frac{1}{2}\sigma^2+\delta+\lambda-e^{(1-\gamma)\nu}\lambda+\gamma\sigma^2-\frac{1}{2}(\gamma\sigma)^2+\bar{\sigma}(B_{t+1}-B_t)+\bar{\nu}(\bar{N}_{t+1}-\bar{N}_t)}\right] \\ &= A_t K_t^{\alpha} e^{-(\alpha\phi+\alpha\delta+\alpha\frac{1}{2}\sigma^2-\delta-\lambda+e^{(1-\gamma)\nu}\lambda-\gamma\sigma^2+\frac{1}{2}(\gamma\sigma)^2-\bar{\mu}-(e^{\bar{\nu}}-1)\bar{\lambda})}, \\ P_{c,t}|_{\rho=\bar{\rho}} &= A_t K_t^{\alpha} e^{\alpha\delta+\alpha\frac{1}{2}\sigma^2-(1-e^{(1-\alpha\gamma)\nu})\lambda-(1-e^{-\gamma\bar{\nu}})\bar{\lambda}-\gamma\alpha\sigma^2+\frac{1}{2}(\gamma\bar{\sigma})^2+\frac{1}{2}(\alpha\gamma\sigma)^2+\bar{\mu}-\frac{1}{2}\bar{\sigma}^2} \\ &\quad \times E_t\left[e^{-\int_t^{t+1}(\frac{\gamma-1}{\gamma}r_s-\delta)ds+(1-\gamma)\left(\bar{\sigma}(B_{t+1}-B_t)+\alpha\sigma(Z_{t+1}-Z_t)+\alpha\nu(N_{t+1}-N_t)+\bar{\nu}(\bar{N}_{t+1}-\bar{N}_t)\right)}\right], \end{aligned}$$

where the latter needs to be determined numerically. Observe that for the case of $\alpha = \gamma$, we obtain the claim of this asset in closed form, with the return (48).

Consider a claim on *capital*

$$R_{c,t+1} = \frac{K_{t+1}^{\alpha\gamma}}{P_{c,t}}, \tag{73}$$

where from (71), we obtain the function of capital as

$$K_s^{\alpha\gamma} = K_t^{\alpha\gamma} e^{\int_t^s (\gamma r_v - \alpha\gamma C_v/K_v - \alpha\gamma\delta - \frac{1}{2}\alpha\gamma\sigma^2)dv + \alpha\gamma\sigma(Z_s - Z_t) + \alpha\gamma\nu(N_s - N_t)}.$$

From (2) we obtain the price of this asset in terms of the consumption good as

$$\begin{aligned}
P_{c,t} &= E_t\left[\frac{m_{t+1}}{m_t} K_{t+1}^{\alpha\gamma}\right] \\
\Rightarrow \quad P_{c,t}|_{\alpha=\gamma} &= K_t^{\alpha\gamma} e^{\delta + \lambda - e^{(1-\gamma)\nu}\lambda + \gamma\sigma^2 - \frac{1}{2}(\gamma\sigma)^2 - \alpha\gamma\phi - \alpha\gamma\delta - \frac{1}{2}\alpha\gamma\sigma^2} \\
&\quad \times E_t\left[e^{-\int_t^{t+1}(1-\gamma)r_s ds + (\alpha-1)\gamma\sigma(Z_s - Z_t) + (\alpha-1)\gamma\nu(N_s - N_t)}\right], \\
P_{c,t}|_{\rho=\bar{\rho}} &= K_t^{\alpha\gamma} e^{-(\alpha\gamma\delta + \frac{1}{2}\alpha\gamma\sigma^2 - \delta - (1 - e^{(1-\alpha\gamma)\nu})\lambda - (1 - e^{-\gamma\bar{\nu}})\bar{\lambda} - \gamma\alpha\sigma^2 + \frac{1}{2}(\gamma\bar{\sigma})^2 + \frac{1}{2}(\alpha\gamma\sigma)^2)} \\
&\quad \times E_t\left[e^{-\gamma\bar{\sigma}(B_{t+1} - B_t) - \gamma\bar{\nu}(\bar{N}_{t+1} - \bar{N}_t)}\right] \\
&= K_t^{\alpha\gamma} e^{-(\alpha\gamma\delta + \frac{1}{2}\alpha\gamma\sigma^2 - \delta - (1 - e^{(1-\alpha\gamma)\nu})\lambda - \gamma\alpha\sigma^2 + \frac{1}{2}(\alpha\gamma\sigma)^2)},
\end{aligned}$$

with the return to the equity claim for the latter yields (49).

Riskless asset. From (41) or (42) and (2), we obtain for any riskless security

$$\begin{aligned}
R_{f,t+1}|_{\alpha=\gamma} &= \left(E_t\left[e^{-\int_t^{t+1}(r_s - \delta)ds + \lambda - e^{(1-\gamma)\nu}\lambda + \gamma\sigma^2 - \frac{1}{2}(\gamma\sigma)^2 - \gamma\sigma(Z_{t+1} - Z_t) - \gamma\nu(N_{t+1} - N_t)}\right]\right)^{-1}, \\
R_{f,t+1}|_{\rho=\bar{\rho}} &= \left(E_t\left[e^{-\int_t^{t+1}(r_s - \delta)ds + (1 - e^{(1-\alpha\gamma)\nu})\lambda + (1 - e^{-\gamma\bar{\nu}})\bar{\lambda} + \gamma\alpha\sigma^2 - \frac{1}{2}(\gamma\bar{\sigma})^2 - \frac{1}{2}(\alpha\gamma\sigma)^2}\right.\right. \\
&\qquad \left.\left. \times e^{-\gamma\bar{\sigma}(B_{t+1} - B_t) - \alpha\gamma\sigma(Z_{t+1} - Z_t) - \alpha\gamma\nu(N_{t+1} - N_t) - \gamma\bar{\nu}(\bar{N}_{t+1} - \bar{N}_t)}\right]\right)^{-1},
\end{aligned}$$

which is not available in closed-form, but will be time-varying.

B.3.6 Euler equation errors

Consider two assets, i.e., the risky bond, $R_{b,t+1}$, and the claim on capital or output, $R_{c,t+1}$.

From the definition of Euler equation errors (3), for any asset i and CRRA preferences

$$\begin{aligned}
e_R^i|_{\alpha=\gamma} &= E_t\left[e^{-\int_t^{t+1}(r_s - \delta)ds + \lambda - e^{(1-\gamma)\nu}\lambda + \gamma\sigma^2 - \frac{1}{2}(\gamma\sigma)^2 - \gamma\sigma(Z_{t+1} - Z_t) - \gamma\nu(N_{t+1} - N_t)} R_{i,t+1}\right] - 1, \\
e_R^i|_{\rho=\bar{\rho}} &= E_t\left[e^{-\int_t^{t+1}(r_s - \delta)ds + (1 - e^{(1-\alpha\gamma)\nu})\lambda + (1 - e^{-\gamma\bar{\nu}})\bar{\lambda} + \gamma\alpha\sigma^2 - \frac{1}{2}(\gamma\bar{\sigma})^2 - \frac{1}{2}(\alpha\gamma\sigma)^2}\right. \\
&\qquad \left. \times e^{-\gamma\bar{\sigma}(B_{t+1} - B_t) - \alpha\gamma\sigma(Z_{t+1} - Z_t) - \alpha\gamma\nu(N_{t+1} - N_t) - \gamma\bar{\nu}(\bar{N}_{t+1} - \bar{N}_t)} R_{i,t+1}\right] - 1,
\end{aligned}$$

where we inserted the SDFs from (41) and (42). Note that Euler equation errors based on excess returns can be obtained from $e_X^i = e_R^i - e_R^b$ for any asset i.

Risky assets. Inserting the one-period equilibrium returns for the bond yields

$$\begin{aligned} e^b_R|_{\alpha=\gamma} &= E_t\left[e^{(1-e^{-\gamma\nu})\lambda-\frac{1}{2}(\gamma\sigma)^2-\gamma\sigma(Z_{t+1}-Z_t)-\gamma\nu(N_{t+1}-N_t)}\right]-1, \\ e^b_R|_{\rho=\bar{\rho}} &= E_t\left[e^{(1-e^{-\alpha\gamma\nu})\lambda+(1-e^{-\gamma\bar{\nu}})\bar{\lambda}-\frac{1}{2}(\gamma\bar{\sigma})^2-\frac{1}{2}(\alpha\gamma\sigma)^2}\right. \\ &\qquad \left.\times e^{-\gamma\bar{\sigma}(B_{t+1}-B_t)-\alpha\gamma\sigma(Z_{t+1}-Z_t)-\alpha\gamma\nu(N_{t+1}-N_t)-\gamma\bar{\nu}(\bar{N}_{t+1}-\bar{N}_t)}\right]-1. \end{aligned}$$

Conditional on no disasters, on average we can rationalize Euler equation errors

$$\begin{aligned} e^b_{R|N_{t+1}-N_t=0}|_{\alpha=\gamma} &= \exp\left((1-e^{-\gamma\nu})\lambda\right)-1, \\ e^b_{R|N_{t+1}-N_t=0}|_{\rho=\bar{\rho}} &= \exp\left((1-e^{-\alpha\gamma\nu})\lambda\right)-1, \end{aligned}$$

or, conditional on no rare events, on average we can rationalize Euler equation errors

$$\begin{aligned} e^b_{R|N_{t+1}-N_t=\bar{N}_{t+1}-\bar{N}_t=0}|_{\alpha=\gamma} &= \exp\left((1-e^{-\gamma\nu})\lambda\right)-1, \\ e^b_{R|N_{t+1}-N_t=\bar{N}_{t+1}-\bar{N}_t=0}|_{\rho=\bar{\rho}} &= \exp\left((1-e^{-\alpha\gamma\nu})\lambda+(1-e^{-\gamma\bar{\nu}})\bar{\lambda}\right)-1. \end{aligned}$$

Similarly, inserting the return on the claims on output (48) and capital (49) yields

$$\begin{aligned} e^c_R|_{\alpha=\gamma} &= E_t\left[e^{-\frac{1}{2}\bar{\sigma}^2-(e^{\bar{\nu}}-1)\bar{\lambda}+\bar{\sigma}(B_{t+1}-B_t)+\bar{\nu}(\bar{N}_{t+1}-\bar{N}_t)}\right]-1, \\ e^c_R|_{\rho=\bar{\rho}} &= E_t\left[e^{-\frac{1}{2}(\gamma\bar{\sigma})^2-(e^{-\gamma\bar{\nu}}-1)\bar{\lambda}-\gamma\bar{\sigma}(B_{t+1}-B_t)-\gamma\bar{\nu}(\bar{N}_{t+1}-\bar{N}_t)}\right]-1, \end{aligned}$$

respectively.

B.3.7 Estimated Euler equation errors

Consider two assets, i.e., the risky bond, $R_{b,t+1}$, and the claim on capital or output, $R_{c,t+1}$.

Using estimated Euler equation errors in (4), for any asset i and CRRA preferences

$$\begin{aligned} \widehat{e^i_R}|_{\alpha=\gamma} &= E_t\left[e^{-\hat{\rho}-(1/\alpha\int_t^{t+1}r_s ds-(\phi+\delta+\frac{1}{2}\sigma^2)+\sigma(Z_{t+1}-Z_t)+\nu(N_{t+1}-N_t))\hat{\gamma}}R_{i,t+1}\right]-1, \\ \widehat{e^i_R}|_{\rho=\bar{\rho}} &= E_t\left[e^{-\hat{\rho}-(1/\gamma\int_t^{t+1}r_s ds+\bar{\mu}-\frac{1}{2}\bar{\sigma}^2-\alpha\delta-\frac{1}{2}\alpha\sigma^2+\bar{\sigma}(B_{t+1}-B_t)+\alpha\sigma(Z_{t+1}-Z_t)+\alpha\nu(N_{t+1}-N_t))\hat{\gamma}}R_{i,t+1}\right]-1, \end{aligned}$$

where we used the equilibrium consumption growth rates from (43) and (44). The estimated Euler equation errors for excess returns can be obtained from $\widehat{e^i_X}=\widehat{e^i_R}-\widehat{e^f_R}$ for any asset i.

Risky assets. Inserting the one-period equilibrium returns for the bond into

$$\begin{aligned} \widehat{e^b_R}|_{\alpha=\gamma} &= E_t\left[e^{-\hat{\rho}-(1/\alpha\int_t^{t+1}r_s ds-(\phi+\delta+\frac{1}{2}\sigma^2)+\sigma(Z_{t+1}-Z_t)+\nu(N_{t+1}-N_t))\hat{\gamma}}R_{b,t+1}\right]-1, \\ \widehat{e^b_R}|_{\rho=\bar{\rho}} &= E_t\left[e^{-\hat{\rho}-(1/\gamma\int_t^{t+1}r_s ds+\bar{\mu}-\frac{1}{2}\bar{\sigma}^2-\alpha\delta-\frac{1}{2}\alpha\sigma^2+\bar{\sigma}(B_{t+1}-B_t)+\alpha\sigma(Z_{t+1}-Z_t)+\alpha\nu(N_{t+1}-N_t))\hat{\gamma}}R_{b,t+1}\right]-1. \end{aligned}$$