FTPL and the maturity structure of government debt in the New-Keynesian Model

Online Appendix

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B Further Results

B.1 Fiscal- and monetary policy shocks

B.1.1 Permanent shocks

Monetary policy. Consider an unexpected decrease in the inflation target by 50 bp, or the policy interest rate target (isomorphic to the inflation target), $i_{ss}^{new} = \rho + \pi_{ss}^{new}$, decreases by 50 bp. Suppose the change is fully credible and observed, i.e., does not require learning or filtering. A lower inflation target then has an expansionary effect on output because it creates inflation and the real interest rate decreases (cf. Figure D.5).

Independent of the maturity structure, the permanent shock does most clearly show up in 10-years ahead inflation expectations and bond yields (cf. Figure D.5). However, the shock is not clearly visible at the shorter end. While the permanent shock increases the 1-year bond yields up to 50 bp, it decreases 10-year bond yields by 50 bp. However, in the model with short-term debt only, the permanent lower inflation target would be even contractionary because lower current inflation increases the real interest rate. Most importantly, the maturity structure matters because the permanent shock even *increases* current expected inflation and decreases the real interest rate (solid blue and black dashed). Because the direct FTPL effect is missing in the model with short-term debt, real debt does not respond immediately and we are left with the indirect effect. However, the direct FTPL effect substantially increases the real value of existing long-term government debt such that the lower inflation target leads to a higher debt-to-GDP ratio, higher tax receipts and thus higher primary surpluses. With short-term debt, the picture is different: initially lower tax revenues (primary surpluses) and lower output with only small changes in real debt lead to negligible effects on the debt-to-GDP ratio. Hence, the maturity effect is more pronounced the longer the average maturity of government debt (cf. Table D.4). In fact, current inflation increases by more than 300 bp in the model with perpetuities with net present value of future inflation of about 8 percent.

How can we understand this dramatic response for inflation dynamics in the model with long-term debt? The simple answer is that the response of inflation is due to a price or valuation effect on existing longer-term bonds, which pay a nominal coupon $\chi + \delta$. Hence, a monetary policy shock in form of a lower inflation target $\pi_t^* \equiv \pi_{ss}^{new} = \pi_{ss} - 0.005$ translates into a higher price $p_{ss}^{b,new}$, and with no change in fiscal surplus results into a lower steady-state value of sovereign debt v_{ss}^{new}. From the decomposition (17), we get

$$\int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u = \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u + p_t^b/p_{ss}^{b,new} - 1 + v_t/v_{ss}^{new} - 1,$$

with a new

$$p_{ss}^{b,new} = \frac{\chi + \delta}{i_{ss}^{new} + \delta}, \quad \text{and} \quad v_{ss}^{new} = a_{ss}/p_{ss}^{b,new}.$$

A permanent monetary policy shock leads to an implicit debt shock $v_t/v_{ss}^{new} - 1$ because of existing longer-term bonds do not sell at the new price p_t^{new} in steady state. Relative to the lower new steady state level of real government debt v_{ss}^{new} (face value), the current debt level v_t now is above its steady-state level – because debt v_t does not jump, which thus can be interpreted as an *implicit* fiscal policy debt shock (compare to Figure 3). This shock is inflationary and the shock size depends on the maturity structure (cf. Table D.4). The effect is already sizable with average maturity (by 2.60 percent), and is substantial with longer maturities (up to more than 11 percent for perpetuities). Both direct effects give the change in the market value of government debt. Even the price effect is negative of about -1.24 percent (p_0^b increases, but p_{ss}^b increases even more), the implied debt shock by 2.60 percent leads to an increase of the market value by 1.36 percent.

C Technical Appendix

C.1 Linearized dynamics

For the alternative parametrizations in Tables D.1 and D.2, the equilibrium dynamics are

$$\mathrm{d}c_t = (i_t - \rho - \pi_t)(1 - s_g)y_{ss}\mathrm{d}t \tag{C.1}$$
$$\mathrm{d}\pi_t = (\rho(\pi_t - \pi_{ss}) - \kappa x_t + \kappa_0 s_g/(1 - s_g)(g_t/g_{ss} - 1))\,\mathrm{d}t \tag{C.2}$$
$$\mathrm{d}i_t = \theta(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_{ss}))\mathrm{d}t \tag{C.3}$$
$$\mathrm{d}a_t = ((i_t - \rho - \pi_t)a_{ss} + \rho(a_t - a_{ss}) - (s_t - s_{ss}))\mathrm{d}t \tag{C.4}$$
$$\mathrm{d}T_t = (\tau_a(a_t - a_{ss}) - (T_t - T_{ss}))\,\mathrm{d}t \tag{C.5}$$
$$\mathrm{d}g_t = (\varphi_a(a_t - a_{ss}) - (g_t - g_{ss}))\,\mathrm{d}t \tag{C.6}$$
$$\mathrm{d}p_t^b = \big((i_t - i_{ss}) + (\chi + \delta)(p_t^b - 1)\big)\,\mathrm{d}t, \tag{C.7}$$

where $s_t = T_t - g_t$, and $\kappa \equiv (1 + \vartheta(1 - s_g))\kappa_0$.

C.2 Term structure of interest rates

The term structure of interest rate reveals important insights into the expectations about the future path of macroeconomic aggregates and inflation. Given the general equilibrium prices (and dynamics), we may price any other asset. For example, a zero-coupon bond with maturity N satisfies the no-arbitrage condition

$$i_t\,\mathrm{d}t = (1/p_t^{(N)})\,\mathrm{d}p_t^{(N)} - (1/p_t^{(N)})(\partial p_t^{(N)}/\partial N)\,\mathrm{d}t,$$

which shows that no-arbitrage considerations imply that the bond prices adjust such that the households will be indifferent in their investment decision. The zero-coupon bonds must inherit the *same* properties as the longer-term bonds introduced above. Hence, since the resulting maturity distribution is approximately exponential with a duration of $1/\delta$, longer-term bonds are interchangeable with zero-coupon bonds with maturity N.[1]

Let us consider a nominal (zero-coupon) bond with unity payoff at maturity N:

$$p_t^{(N)} = \mathbb{E}_t\left(e^{-\rho N}\lambda_{t+N}/\lambda_t e^{-\int_t^{t+N}\pi_s ds}\right),$$

where λ_t is the marginal value of wealth, or the current value shadow price, consistent with equilibrium dynamics of macro aggregates. The equilibrium bond price can be obtained

[1] Alternatively, consider the bond with nominal coupon payment in (7). The equilibrium price p_t^b can be computed along the same lines from the no-arbitrage condition as a function of the state variables.

from the fundamental pricing equation for the price $p_t^{(N)}$ (cf. Posch, 2020):

$$\mathbb{E}_t\left((\mathrm{d}p_t^{(N)})/p_t^{(N)}\right) - \left(1/p_t^{(N)}(\partial p_t^{(N)}/\partial N) + i_t\right)\,\mathrm{d}t = 0.$$

Observe that in equilibrium, the bond price $p_t^{(N)}$ is a function of the state variables, so $p_t^{(N)} = p_t^{(N)}(i_t, a_t, T_t, g_t)$, where from (14c), (14d), and (14e) we get

$$\begin{aligned}
\mathrm{d}p_t^{(N)} &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t)\,\mathrm{d}t \\
&\quad +(\partial p_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + (\partial p_t^{(N)}/\partial T_t)(\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\,\mathrm{d}t \\
&\quad +(\partial p_t^{(N)}/\partial g_t)(\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\,\mathrm{d}t
\end{aligned}$$

and thus the partial differential equation (henceforth *PDE approach*) is:

$$\begin{aligned}
&(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t) + (\tau_a(a_t - a_{ss}) - (T_t - T_t^*))(\partial p_t^{(N)}/\partial T_t) \\
&+((i_t - \pi_t)a_t - s_t)(\partial p_t^{(N)}/\partial a_t) + (\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))(\partial p_t^{(N)}/\partial g_t) \\
&= (\partial p_t^{(N)}/\partial N) + i_t p_t^{(N)}.
\end{aligned}$$

The solution to the pricing equation implies the complete term structure of interest rate for any given interest rate and maturity:

$$y_t^{(N)} \equiv y^{(N)}(i_t, a_t, T_t, g_t) = -\log p_t^{(N)}(i_t, a_t, T_t, g_t)/N.$$

Our strategy is to use collocation to approximate the function $p_t^{(N)} \approx \Phi(N, i_t, a_t, T_t, g_t)v$, in which v is an n-vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a *linear* interpolation equation:

$$\begin{aligned}
&(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2'(N, i_t, a_t, T_t, g_t)v + ((i_t - \pi_t)a_t - s_t)\Phi_3'(N, i_t, a_t, T_t, g_t)v \\
&+(\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\Phi_4'(N, i_t, a_t, T_t, g_t)v \\
&+(\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\Phi_5'(N, i_t, a_t, T_t, g_t)v \\
= \;& \Phi_1'(N, i_t, a_t, T_t, g_t)v + i_t\Phi(N, i_t, a_t, T_t, g_t)v
\end{aligned}$$

or

$$\begin{aligned}
&\big((\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\Phi_4' + (\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\Phi_5' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\
&\quad +(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1' - i_t\Phi\big)v = 0_n,
\end{aligned}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4 \cdot n_5$ with the boundary condition $\Phi(0, i_t, a_t, T_t, g_t)v = 1_n$. So we concatenate the two matrices and solve the linear equation for the unknown coefficients.

C.3 Expected inflation

Inflation expectations are at the core of monetary policy, so it is useful to study the effects of monetary and fiscal policy shocks on the model-implied expected inflation as a benchmark within our framework. From the Phillips curve in (14b) it follows

$$\pi_t - \pi_t^* = \kappa \int_t^\infty e^{-\rho(v-t)} x_v \mathrm{d}v.$$

The inflation rate, π_t, denotes *current* expected inflation measured as deviation from its policy target rate π_t^*. Multiplying the differential equation for the inflation rate by the integrating factor and evaluating from t to $t+N$, we obtain

$$\pi_t^{(N)} \equiv \mathbb{E}_t(\pi_{t+N}) = \pi_t^* + e^{\rho N}(\pi_t - \pi_t^*) - \kappa e^{\rho N} \int_t^{t+N} e^{-\rho(s-t)} x_s \,\mathrm{d}s.$$

The rational expectation forecast π_{t+N} can be regarded as a function of the state variables (i_t, a_t, T_t and g_t) as well as the fixed forecasting horizon N. Hence, for the N-year ahead future expected inflation rate, we compute $\pi_t^{(N)}$

$$\begin{aligned}
\partial\pi_t^{(N)}/\partial N \;=\; & (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial\pi_t^{(N)}/\partial i_t)\,\mathrm{d}t + (\partial\pi_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t \\
& + (\partial\pi_t^{(N)}/\partial T_t)(\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\,\mathrm{d}t \\
& + (\partial\pi_t^{(N)}/\partial g_t)(\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\,\mathrm{d}t
\end{aligned}$$

with boundary condition $\pi_t^{(0)} = \pi_t^{(0)}(i_t, a_t, T_t, g_t) = \pi_t$. The solution implies the term structure of N-years ahead inflation expectations for a given state $\pi_t^{(N)} = \pi_t^{(N)}(i_t, a_t, T_t, g_t)$. Our strategy is to use collocation to approximate the function $\pi_t^{(N)} \approx \Phi(N, i_t, a_t, T_t, g_t)v$. The n-vector v is a vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a linear interpolation equation

$$\begin{aligned}
& \big((\tau_a(a_t - a_{ss}) - (T_t - T_t^*))\Phi_4' + (\varphi_a(a_t - a_{ss}) - (g_t - g_t^*))\Phi_5' \\
& + ((i_t - \pi_t)a_t - s_t)\Phi_3' + (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1'\big)v = 0_n,
\end{aligned}$$

using the solution to the FTPL-NK model (14a) to (14e) above.

D Tables and Figures

Table D.1: Parametrization 2 (cf. Kliem et al., 2016)

ρ	0.03	subjective rate of time preference, $\rho = -4 \log 0.9925$
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady-state output
ϑ	1	Frisch labor supply elasticity
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	0	output response fiscal tax rule
τ_a	0.01	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	0	output response fiscal expenditure rule
φ_a	-0.01	debt response fiscal expenditure rule
ρ_g	1	inertia of fiscal expenditure rule
s_g	0.1534	output share for government expenditures (Bilbiie, Monacelli, and Perotti (2019))
s_{ss}	0.0324	steady-state primary surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

Table D.2: Parametrization 3 (similar to Sims, 2011; Cochrane, 2018)

ρ	0.03	subjective rate of time preference
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady state output
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	1	output response fiscal tax rule (Sims, 2011; Cochrane, 2018)
τ_a	0	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	-0.1534	output response fiscal expenditure rule
φ_a	0	debt response fiscal expenditure rule
ρ_g	1	inertia of fiscal expenditure rule
s_g	0.1534	government consumption to output ratio (Bilbiie et al., 2019)
s_{ss}	0.0324	steady-state surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

D.1 Policy functions

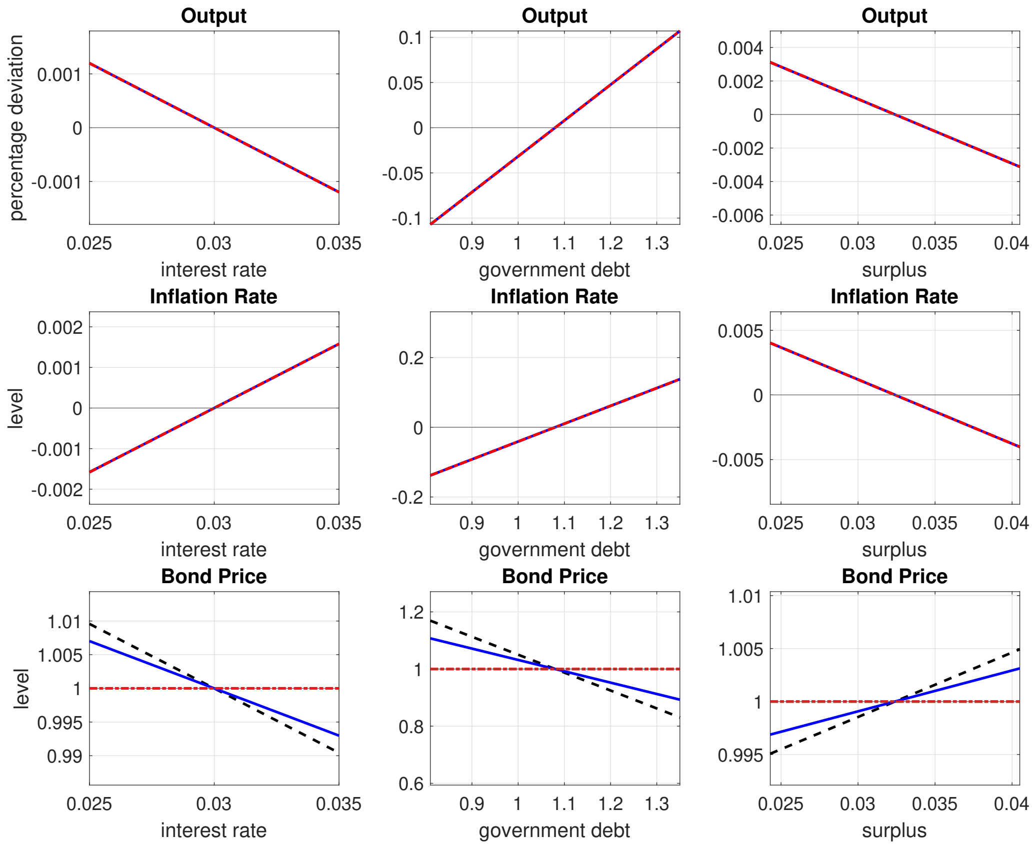

Figure D.1: Policy functions for the parametrization in Table 1, showing the partial response in terms of a_t (indirect effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

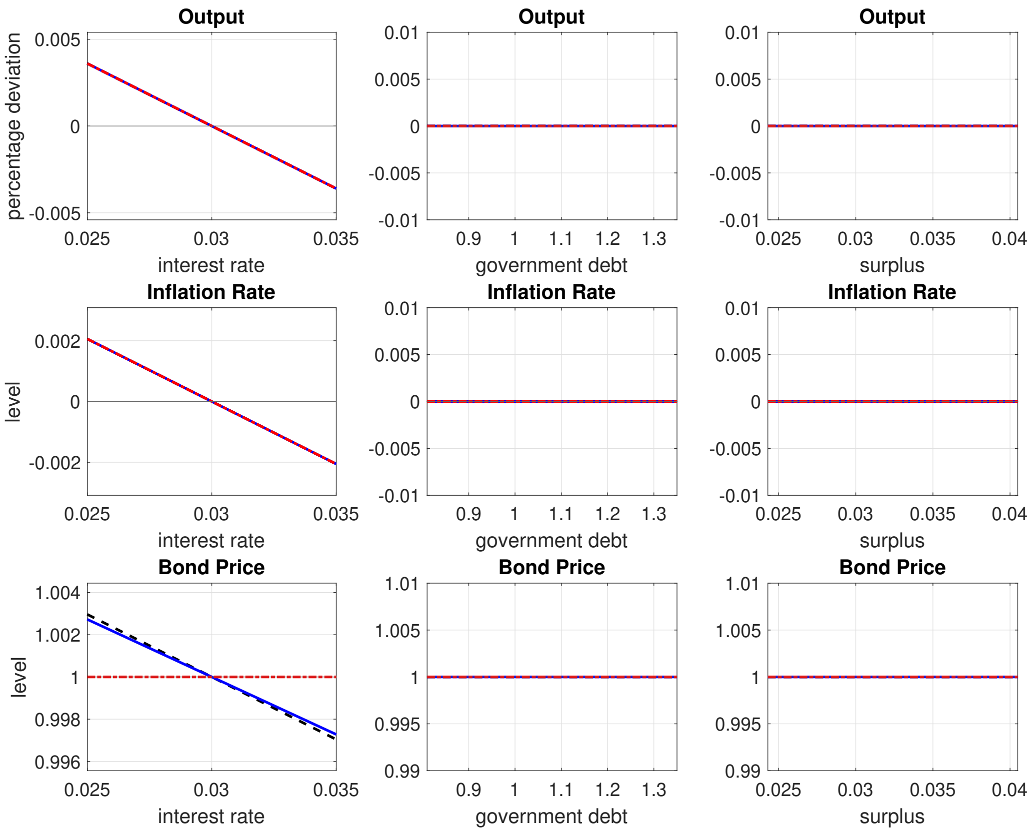

Figure D.2: Policy functions for the parametrization in Table 1, $\phi_\pi = 1.6$ and $\tau_a = 0.04$ (active monetary/passive fiscal policy) in terms of v_t (or a_t). Solid blue for average duration, dashed black for perpetuities, and dotted red for short-term debt.

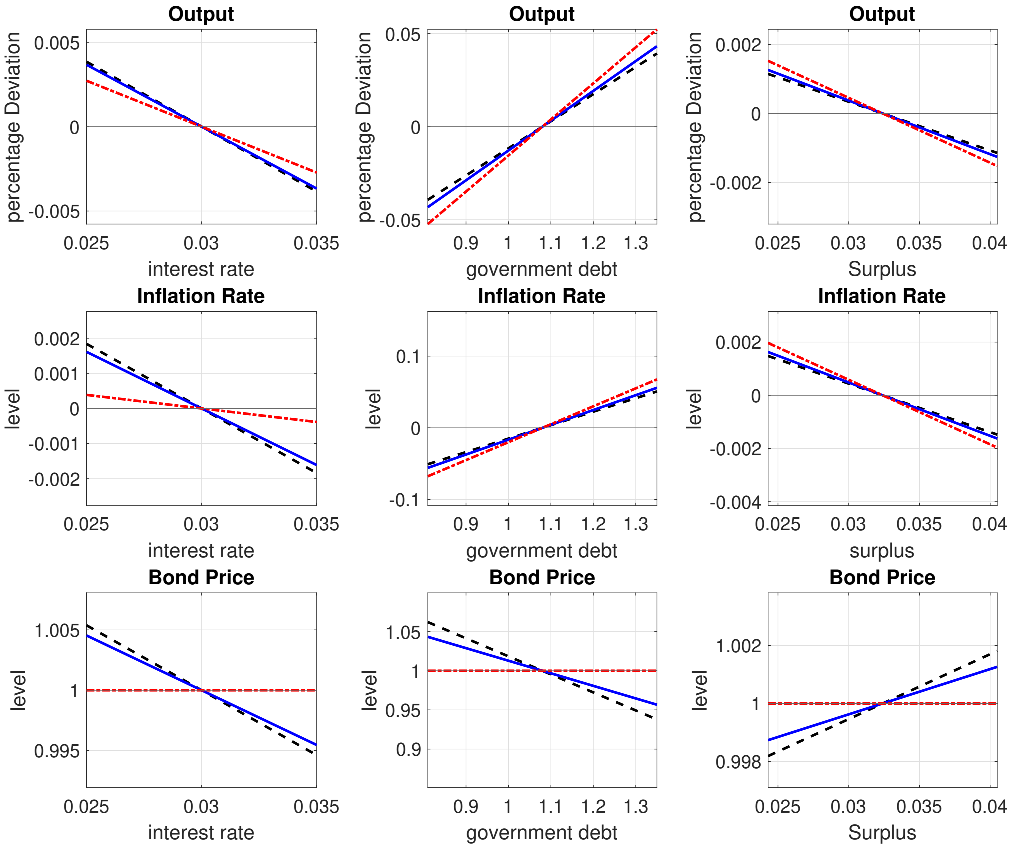

Figure D.3: Policy functions for the parametrization in Table 1 together with an explicit inflation response in (23) in terms of v_t. Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

D.2 Transitional dynamics

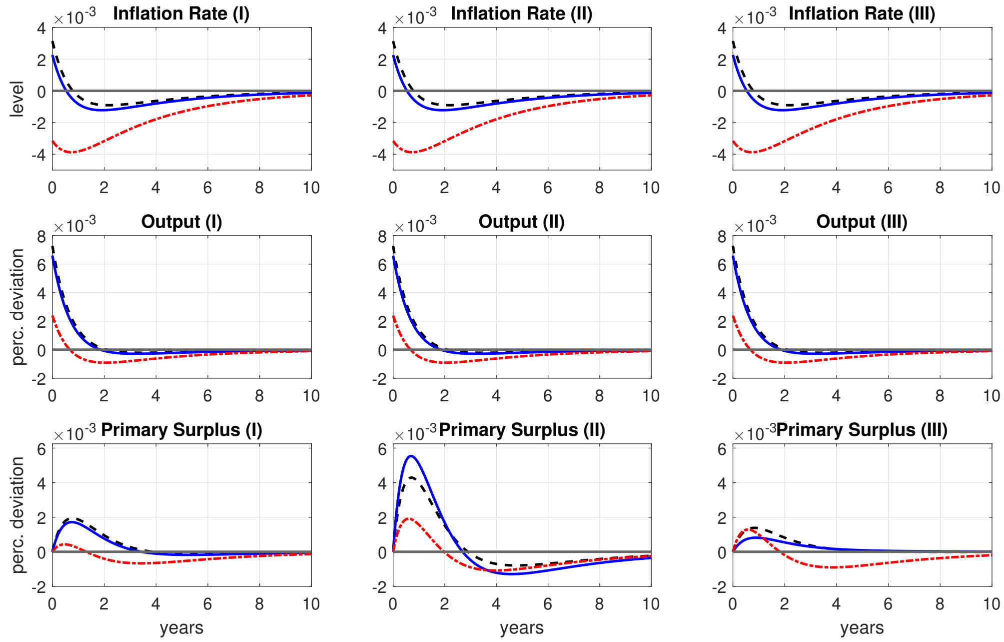

Figure D.4: Transitory monetary policy shock for the parametrization in Table 1 and different surplus dynamics. Decrease nominal interest rate by 1 percentage point. Left-hand panel: Baseline scenario, $\tau_\pi = 0$ and $\tau_y = 1$. Middle panel: $\tau_\pi = 1.02$ and $\tau_y = 3.08$. Right-hand panel: $\tau_\pi = 0.5$ and $\tau_y = -0.25$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.3: Inflation decomposition (17) for the monetary policy shock in Figure D.4.

Surplus Rule	Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
I	Average	-0.48	-1.25	0.21	0.98
II	Average	-0.48	-1.25	0.21	0.98
III	Average	-0.48	-1.25	0.21	0.98

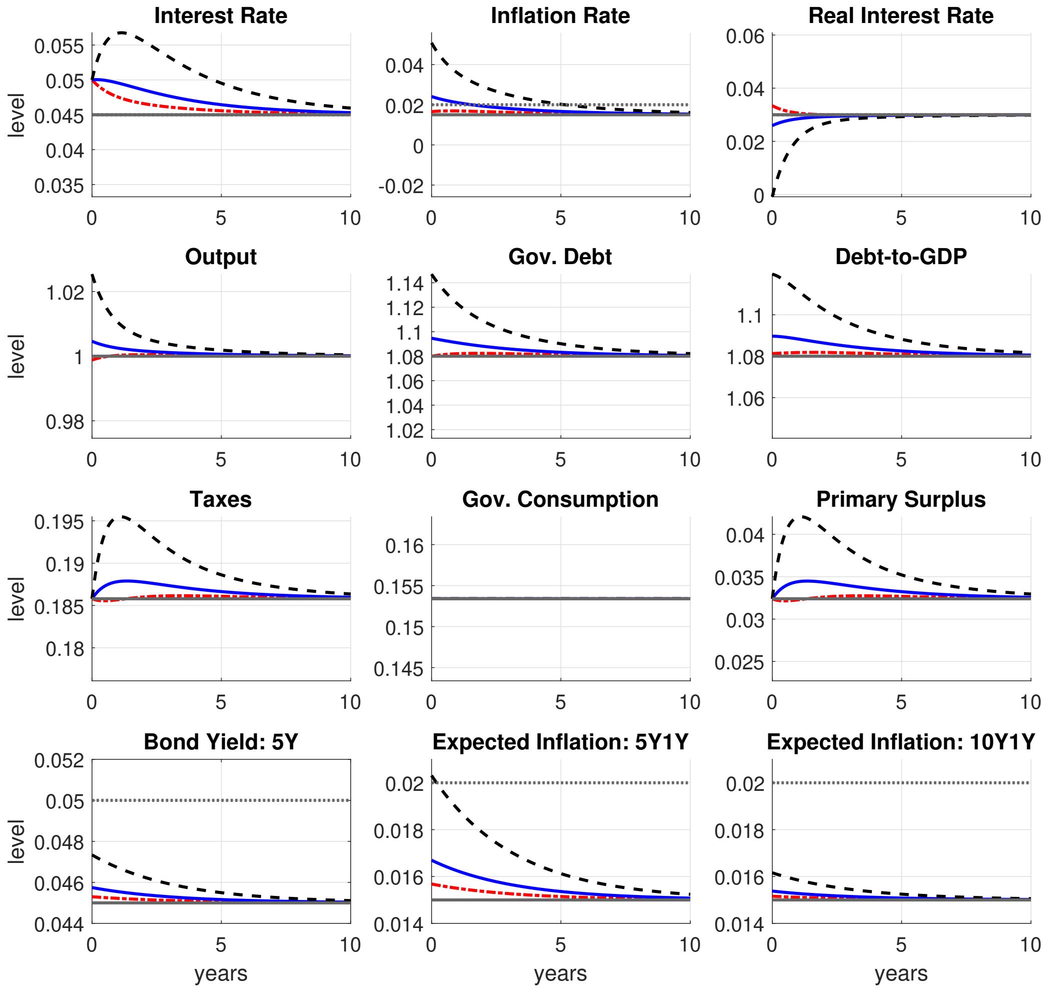

Figure D.5: Permanent monetary policy shock for the parametrization in Table 1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.4: Inflation decomposition (17) for the target rate shock in Figure D.5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^{b,new} - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.02	5.16	3.34	−4.91	11.11
Average	2.39	1.88	0.85	−1.24	2.60
Short-Term	0.81	0.96	0.15	0	0

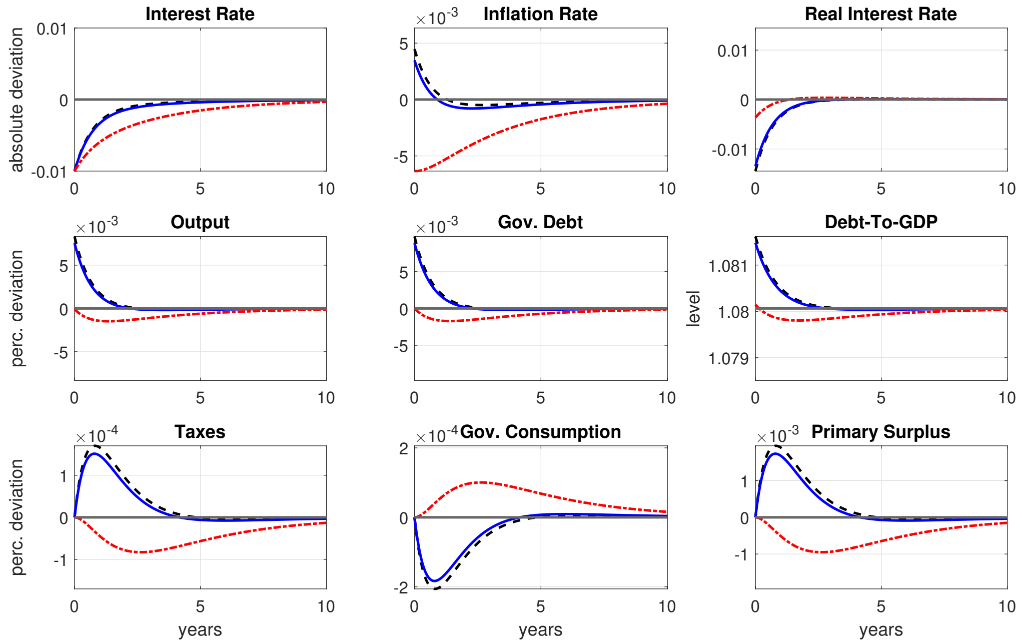

Figure D.6: Transitory monetary policy shock for the parametrization in Table D.1. Decrease in interest rate by 100 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

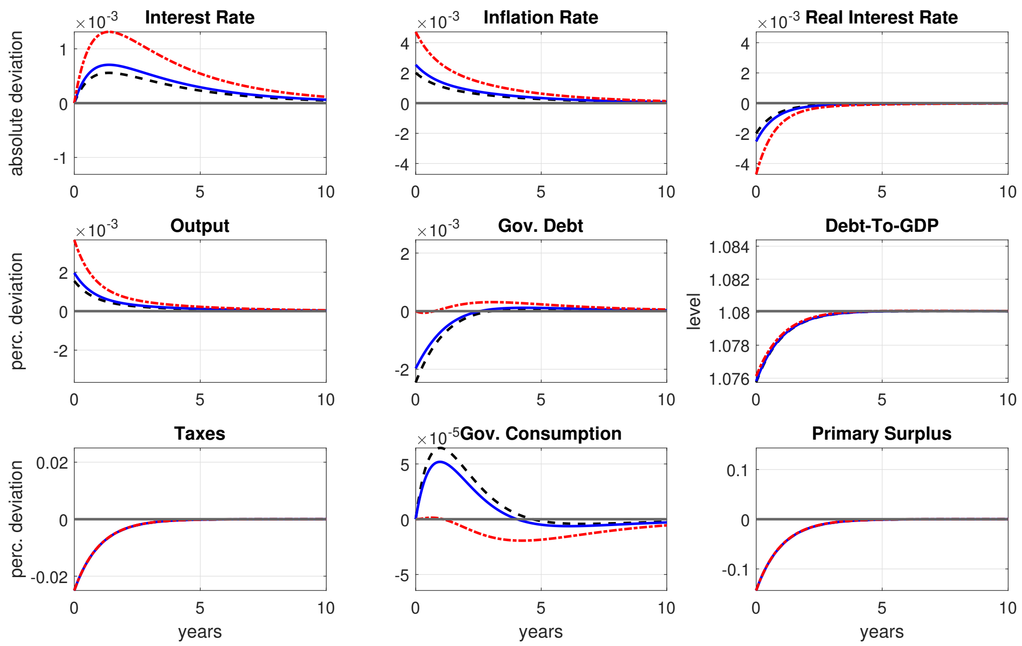

Figure D.7: Transitory fiscal policy shock for the parametrization in Table D.1. Decrease in taxes by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

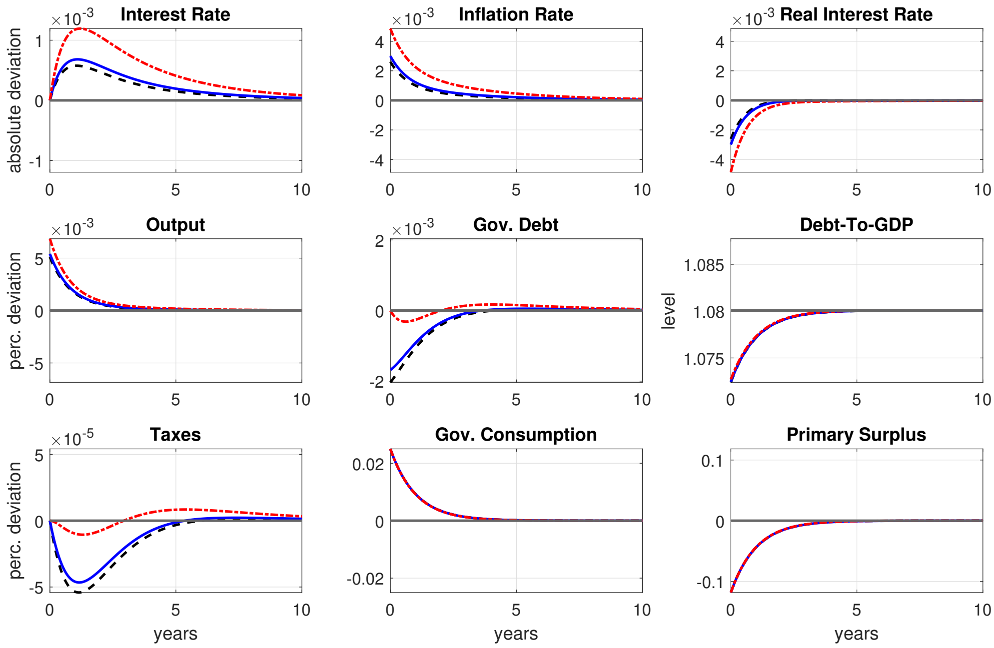

Figure D.8: Transitory fiscal policy shock for the parametrization in Table D.1. Increase in government consumption by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

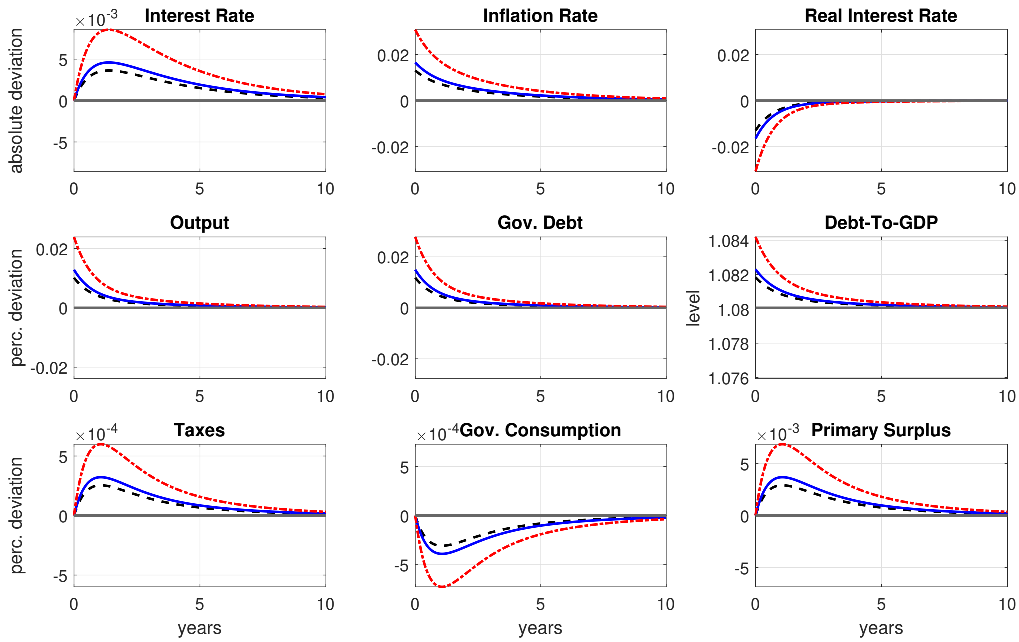

Figure D.9: Transitory fiscal policy shock for the parametrization in Table D.1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

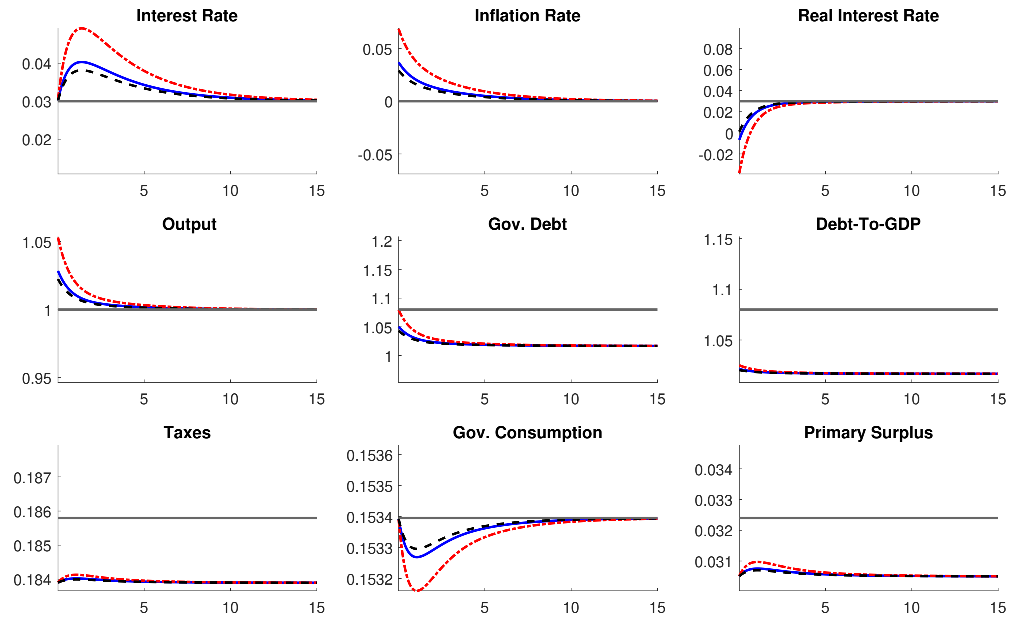

Figure D.10: Permanent fiscal policy shock for parametrization in Table D.1. Permanent decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$, together with a transitory shock that decreases taxes by 1 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

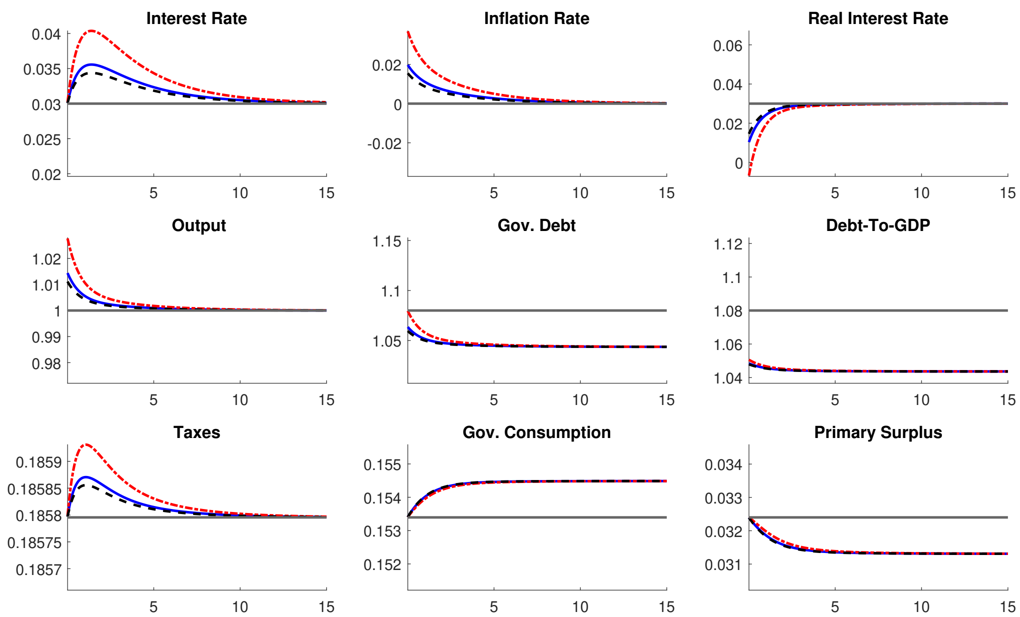

Figure D.11: Permanent fiscal policy shock for the parametrization in Table D.1. Increase of g_{ss} by 1 percent to $g_{ss}^{new} = 1.01g_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

D.3 Inflation and output decomposition

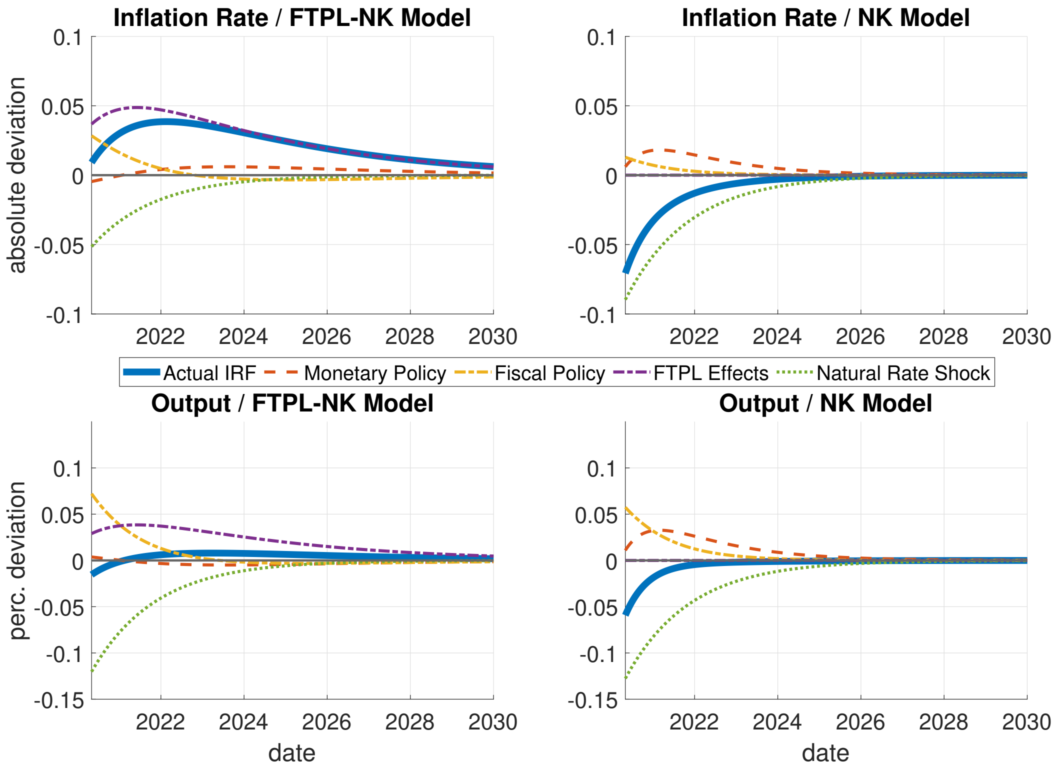

Figure D.12: Decomposition of inflation and output as proposed in Proposition 1 for the CARES Act Shock with interest rate cut by 150 bp. Left-hand side panels: FTPL-NK model. Right-hand side panels: Simple NK model. Upper panels: Inflation dynamics with Monetary Policy $\equiv \bar{\pi}_i(i_t - i_{ss})$, Fiscal Policy $\equiv \bar{\pi}_g(g_t - g_{ss}) + \bar{\pi}_T(T_t - T_{ss})$, FTPL Effects $\equiv \bar{\pi}_a(a_t - a_{ss})$, and Natural Rate Shock $\equiv \bar{\pi}_d(d_t - d_{ss})$. Lower panels: Output dynamics with Monetary Policy $\equiv \bar{x}_i(i_t - i_{ss})$, Fiscal Policy $\equiv (1 + \bar{x}_g)(g_t - g_{ss}) + \bar{x}_T(T_t - T_{ss})$, FTPL Effects $\equiv \bar{x}_a(a_t - a_{ss})$, and Natural Rate Shock $\equiv \bar{x}_d(d_t - d_{ss})$.

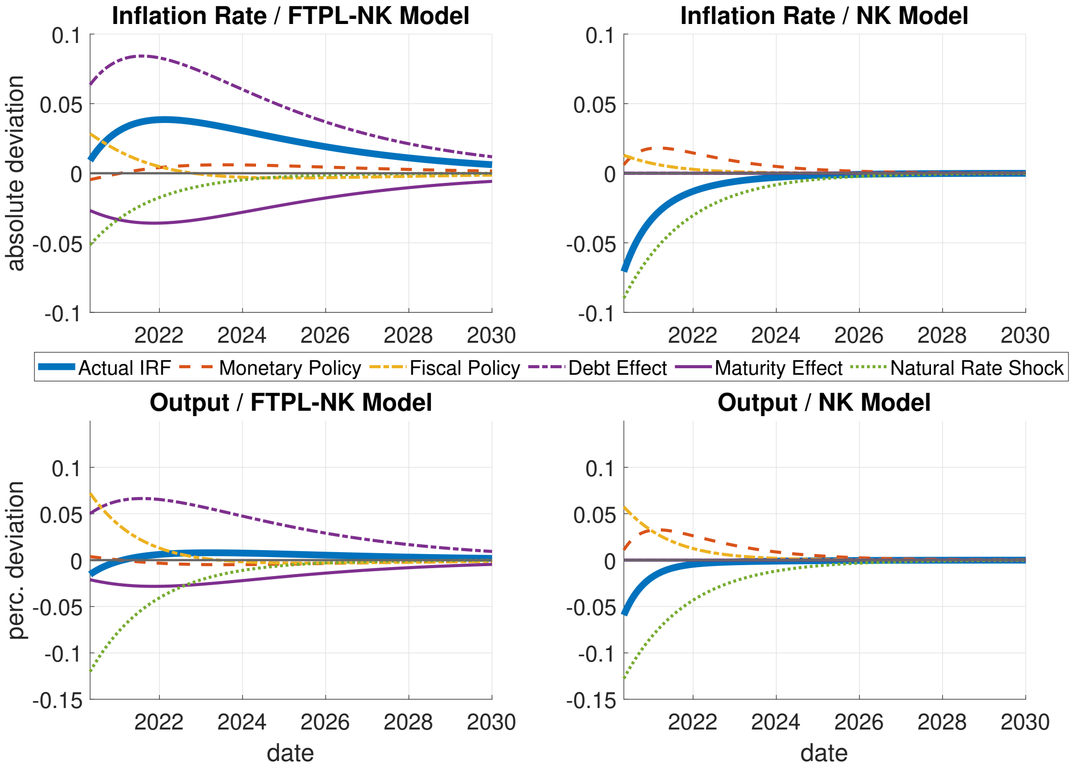

Figure D.13: Decomposition of inflation and output as proposed in Proposition 1 for the CARES Act Shock with interest rate cut by 150 bp. Left-hand site panels: FTPL-NK model. Right-hand site panels: Simple NK model. Upper panels: Inflation dynamics with Monetary Policy $\equiv \bar{\pi}_i(i_t - i_{ss})$, Fiscal Policy $\equiv \bar{\pi}_g(g_t - g_{ss}) + \bar{\pi}_T(T_t - T_{ss})$, Debt Effect $\equiv \bar{\pi}_a p^b_{ss}(v_t - v_{ss})$, Maturity Effect $\equiv \bar{\pi}_a v^b_{ss}(p^b_t - p^b_{ss})$, and Natural Rate Shock $\equiv \bar{\pi}_d(d_t - d_{ss})$. Lower panels: Output dynamics with Monetary Policy $\equiv \bar{x}_i(i_t - i_{ss})/y_{ss}$, Fiscal Policy $\equiv (1 + \bar{x}_g)(g_t - g_{ss})/y_{ss} + \bar{x}_T(T_t - T_{ss})$, Debt Effect $\equiv \bar{x}_a(v_t - v_{ss})$, Maturity Effect $\equiv \bar{x}_a v^b_{ss}(p^b_t - p^b_{ss})$, and Natural Rate Shock $\equiv \bar{x}_d(d_t - d_{ss})$.

D.4 Counterfactual analysis

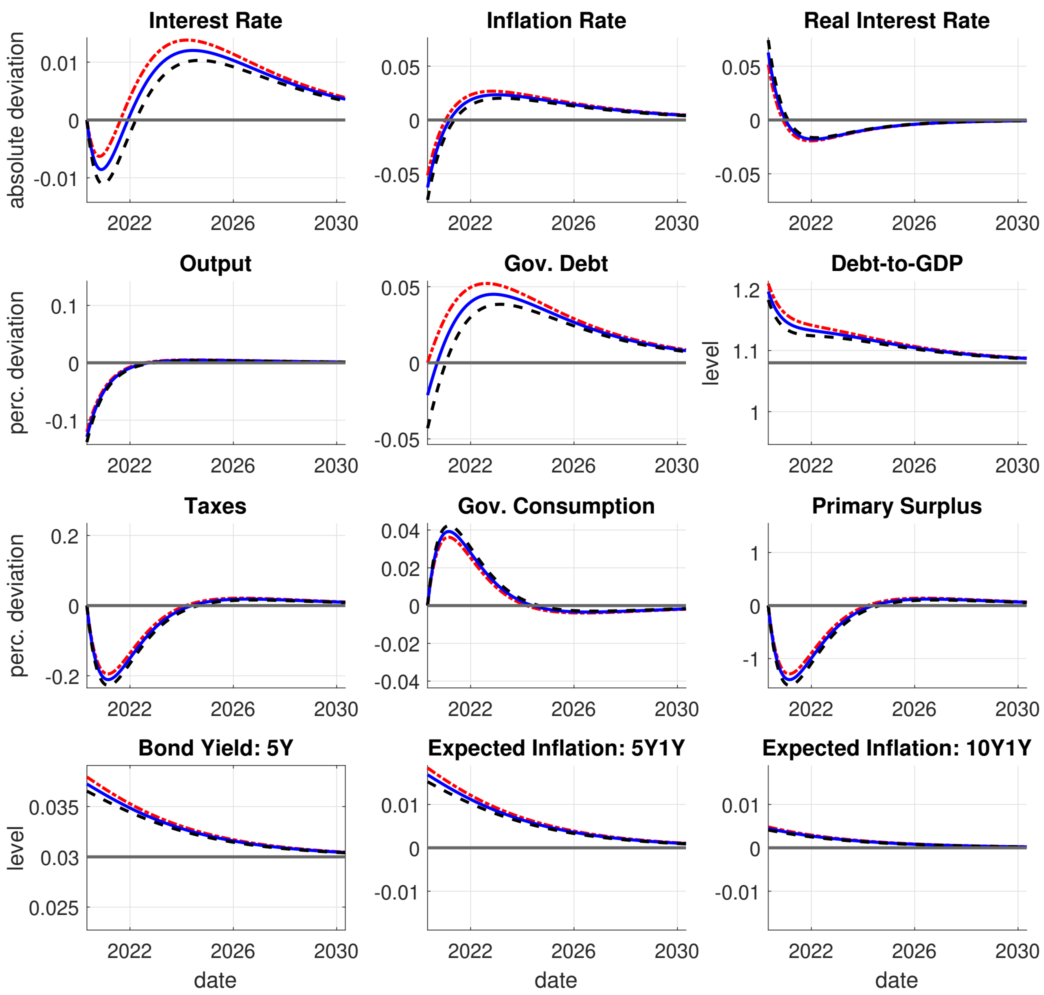

Figure D.14: Counterfactual dynamics: No CARES Act for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.5: Inflation decomposition (17) for the natural rate shock in Figure D.14.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	7.39	4.30	−7.39	−4.30	0
Average	9.85	5.74	−6.24	−2.13	0
Short-Term	12.26	7.14	−5.12	0	0

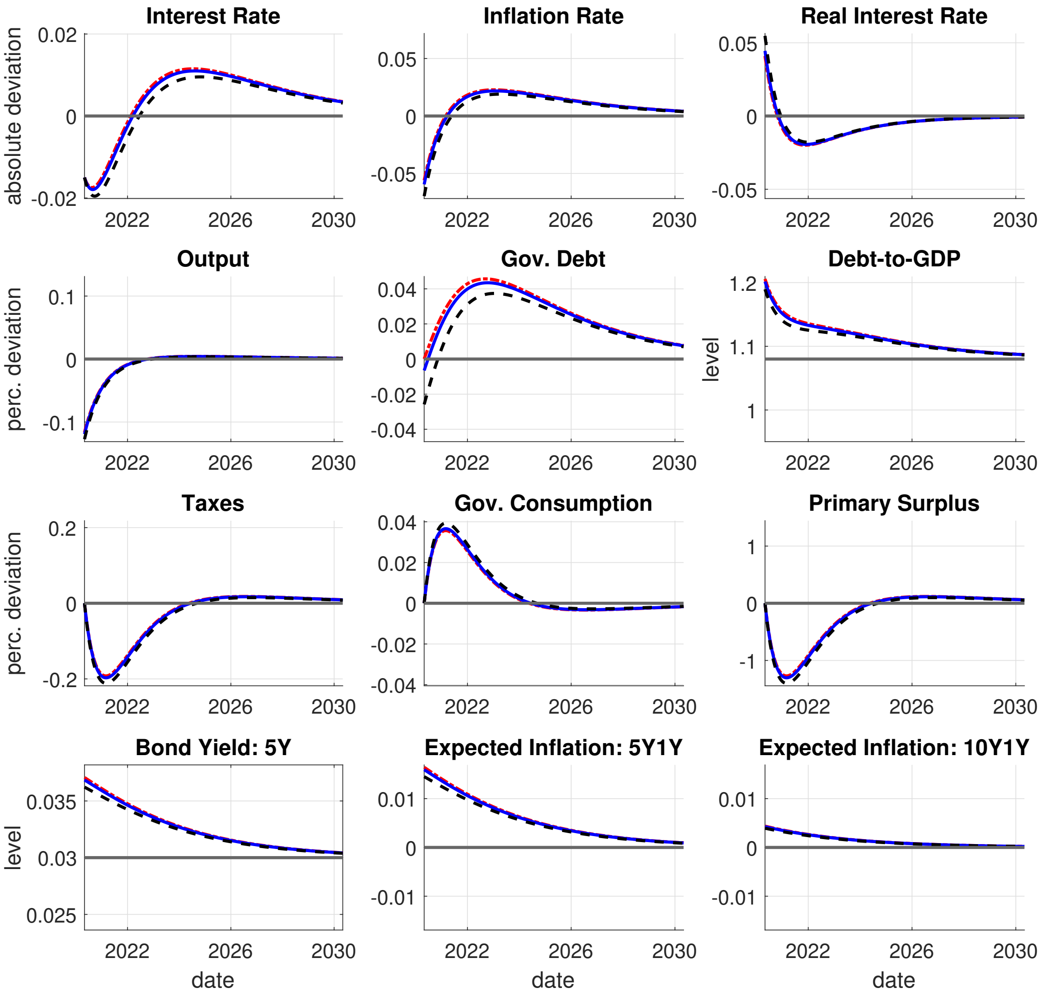

Figure D.15: Counterfactual dynamics: No CARES Act with interest rate cut by 150 bp for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.6: Inflation decomposition (17) for the shocks in Figure D.15.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	6.94	2.59	−6.94	−2.59	0
Average	9.13	3.86	−5.92	−0.65	0
Short-Term	9.87	4.29	−5.57	0	0

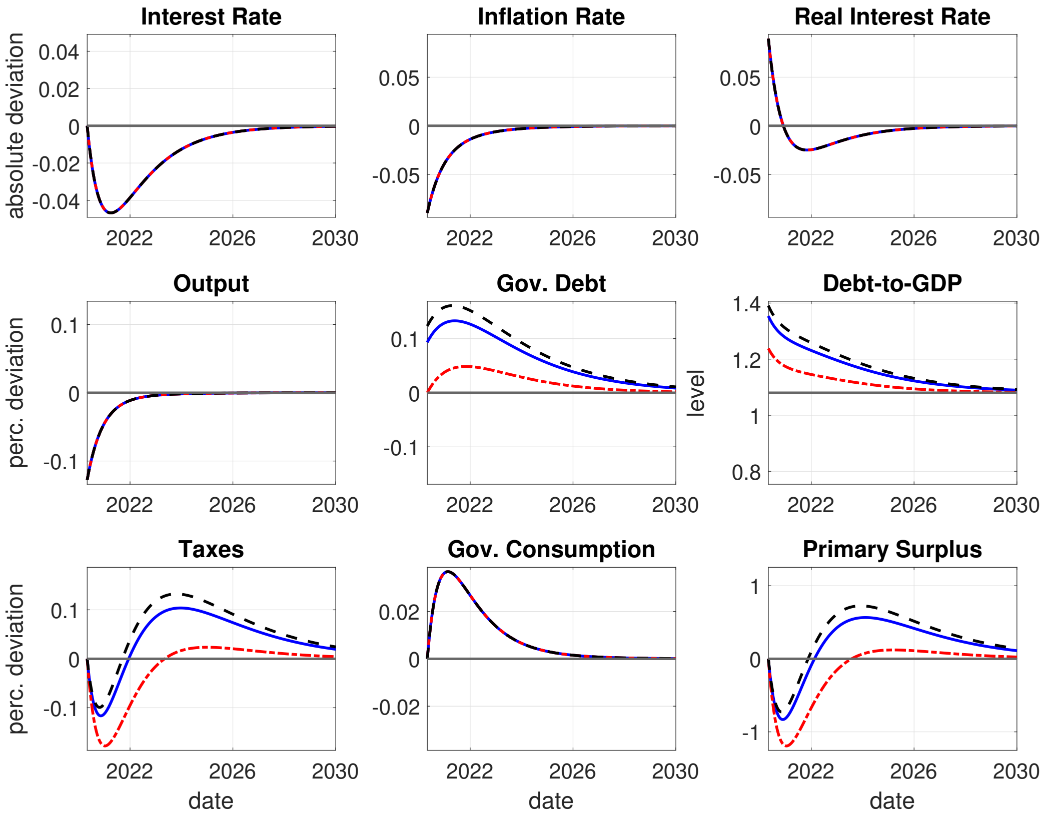

Figure D.16: Counterfactual dynamics: No CARES Act for the parametrization in Table 1 with $\rho_g = 1$, $\varphi_y = -s_g$ and active monetary policy with $\phi_\pi = 1.6$ and $\tau_a = 0.25$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.7: Inflation decomposition (17) for the natural rate shock in Figure D.16.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	-7.94	-12.33	7.94	12.33	0
Average	-7.94	-12.33	4.94	9.33	0
Short-Term	-7.94	-12.33	-4.39	0	0

D.5 Robustness results

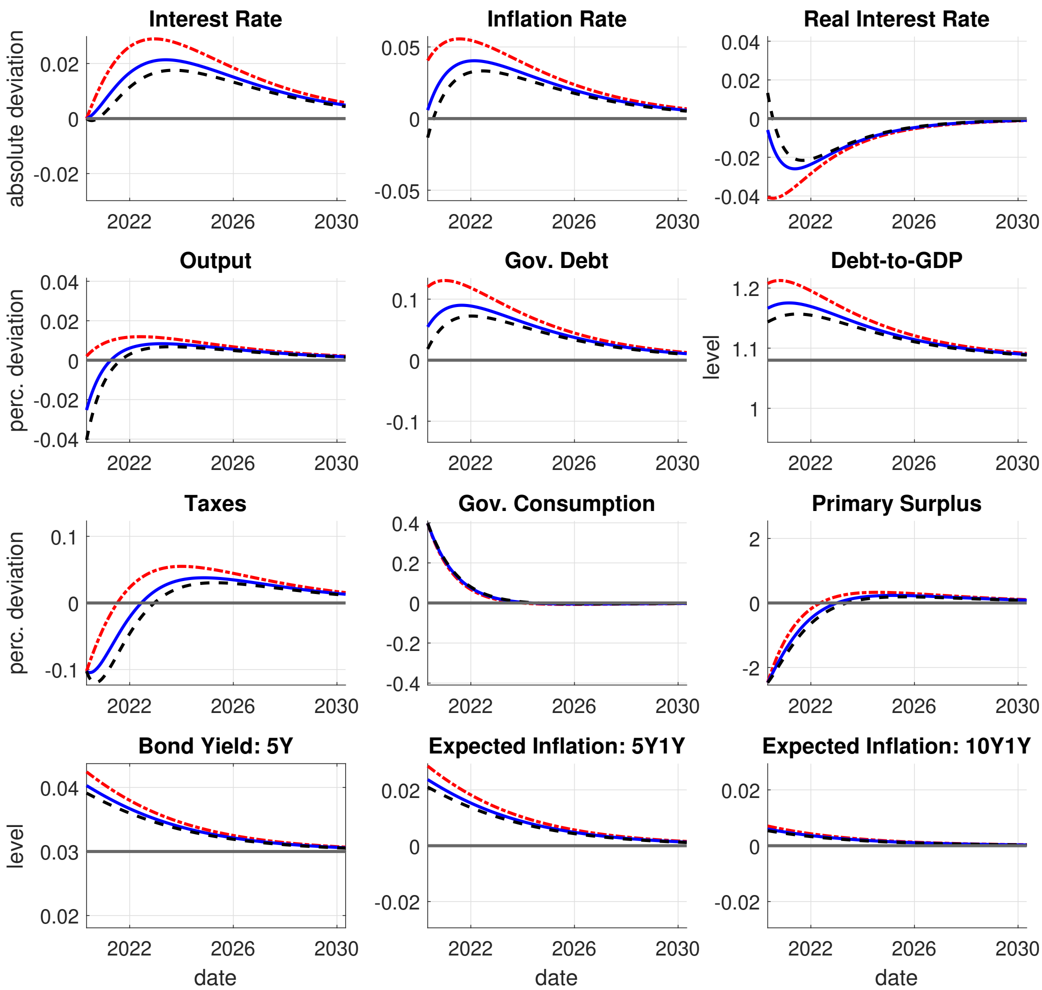

Figure D.17: Transitory CARES Act shock for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.8: Inflation decomposition (17) for the CARES Act shock in Figure D.17.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	17.44	10.16	−5.44	−10.16	12.00
Average	21.54	12.55	−3.53	−6.54	12.00
Short-Term	28.94	16.86	−0.08	0	12.00

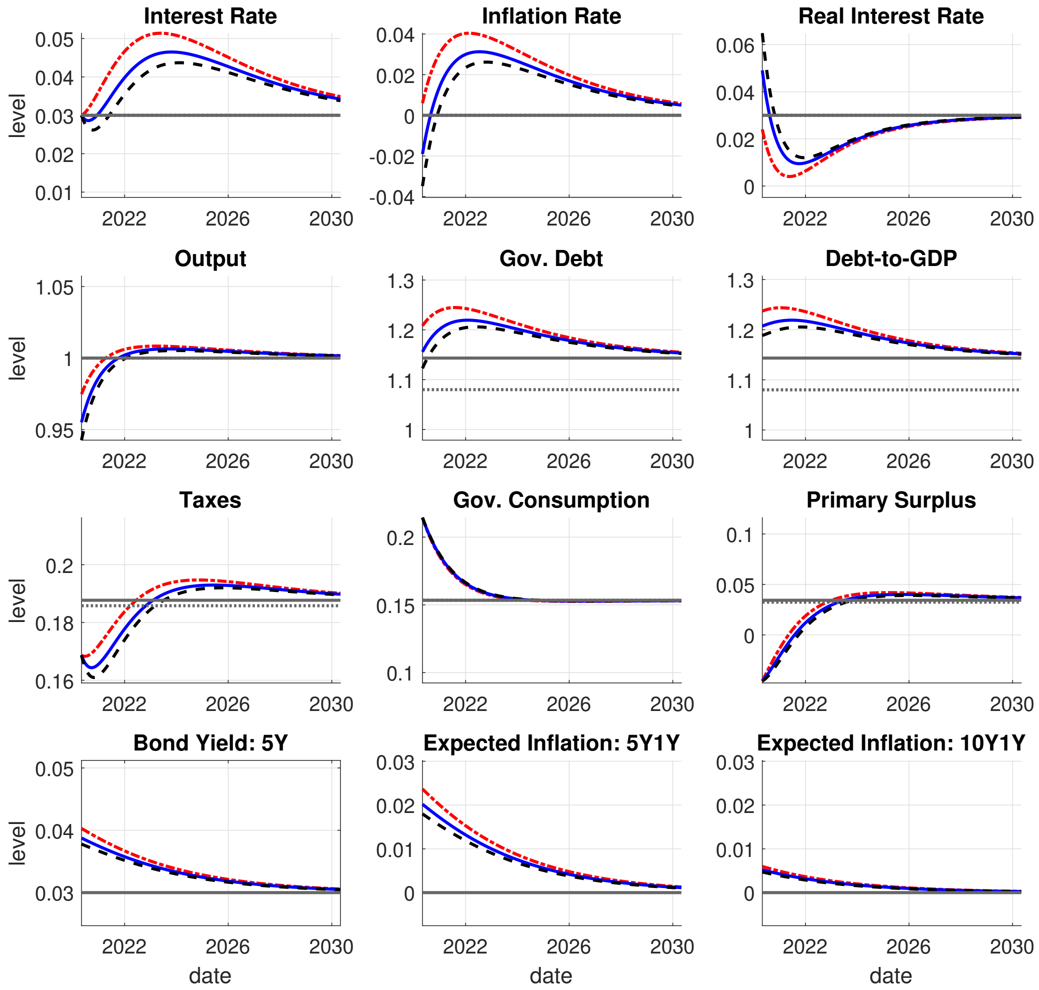

Figure D.18: CARES Act shock with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.9: Inflation decomposition (17) for the CARES Act shock in Figure D.18.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	12.83	7.47	−7.16	−7.47	5.67
Average	16.21	9.44	−5.67	−4.58	5.67
Short-Term	21.55	12.55	−3.32	0	5.67

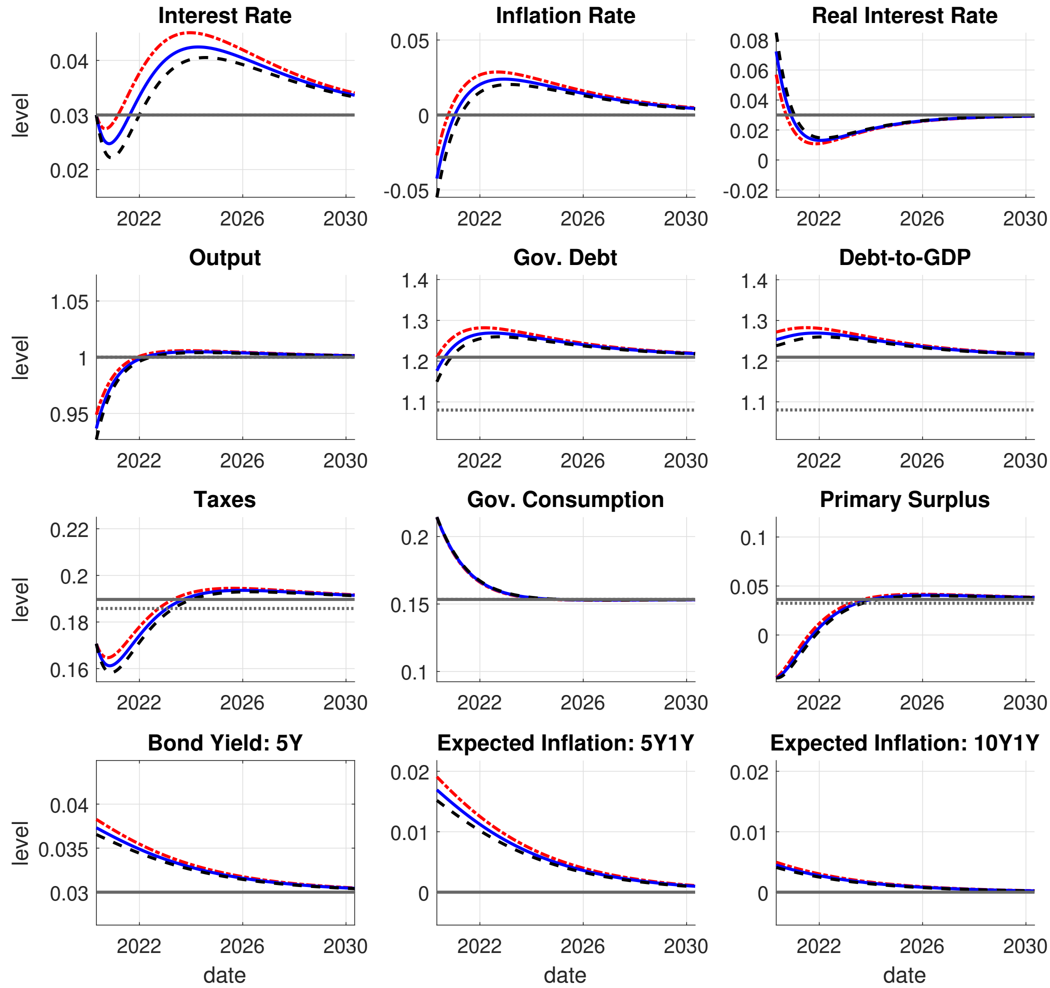

Figure D.19: CARES ACT shock with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.10: Inflation decomposition (17) for the CARES Act shock in Figure D.19.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u / a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.55	4.98	−8.55	−4.98	0
Average	11.23	6.54	−7.44	−2.75	0
Short-Term	14.52	8.46	−6.06	0	0

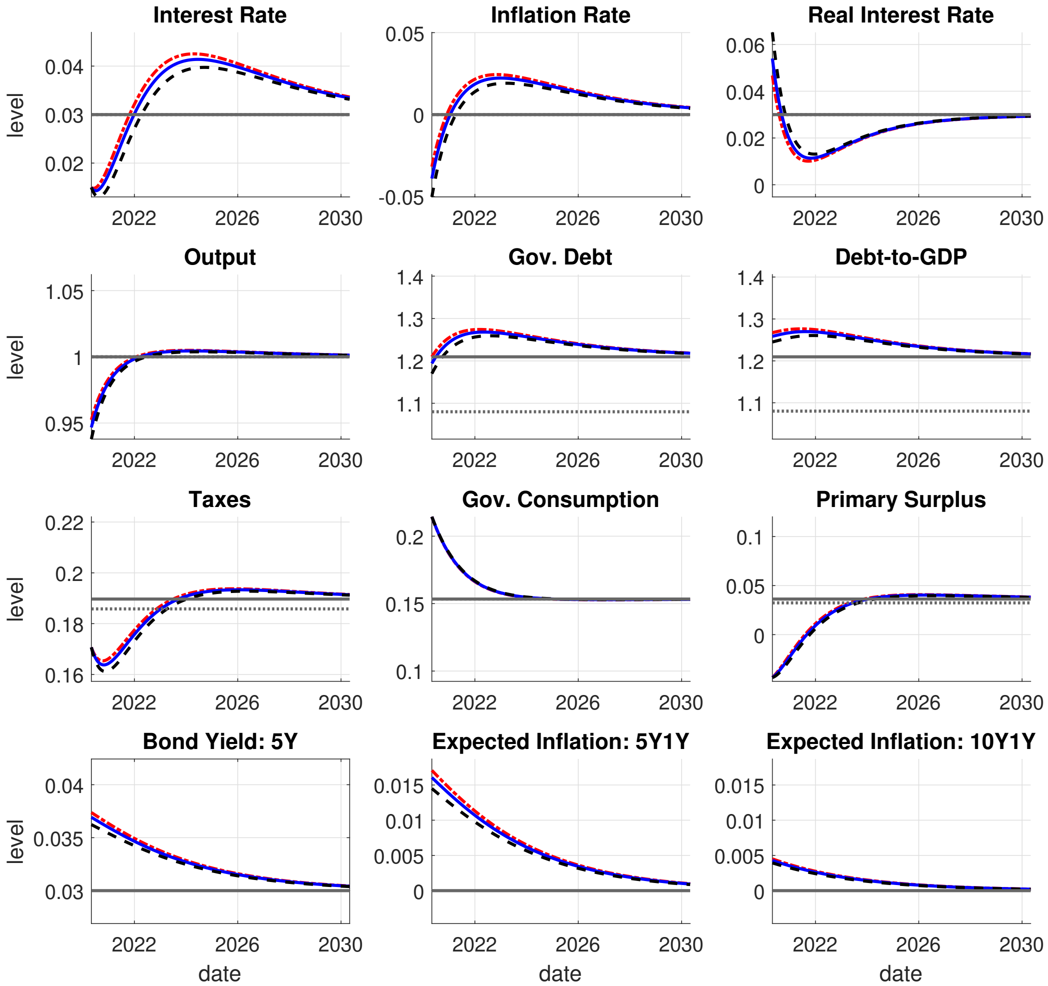

Figure D.20: CARES ACT shock with monetary policy shock and with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, increase in debt (face value) by 12 percent and interest rate cut by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table D.11: Inflation decomposition (17) for the CARES Act shock in Figure D.20.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u / a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.14	3.28	−8.13	−3.28	0
Average	10.53	4.68	−7.14	−1.29	0
Short-Term	12.08	5.58	−6.50	0	0

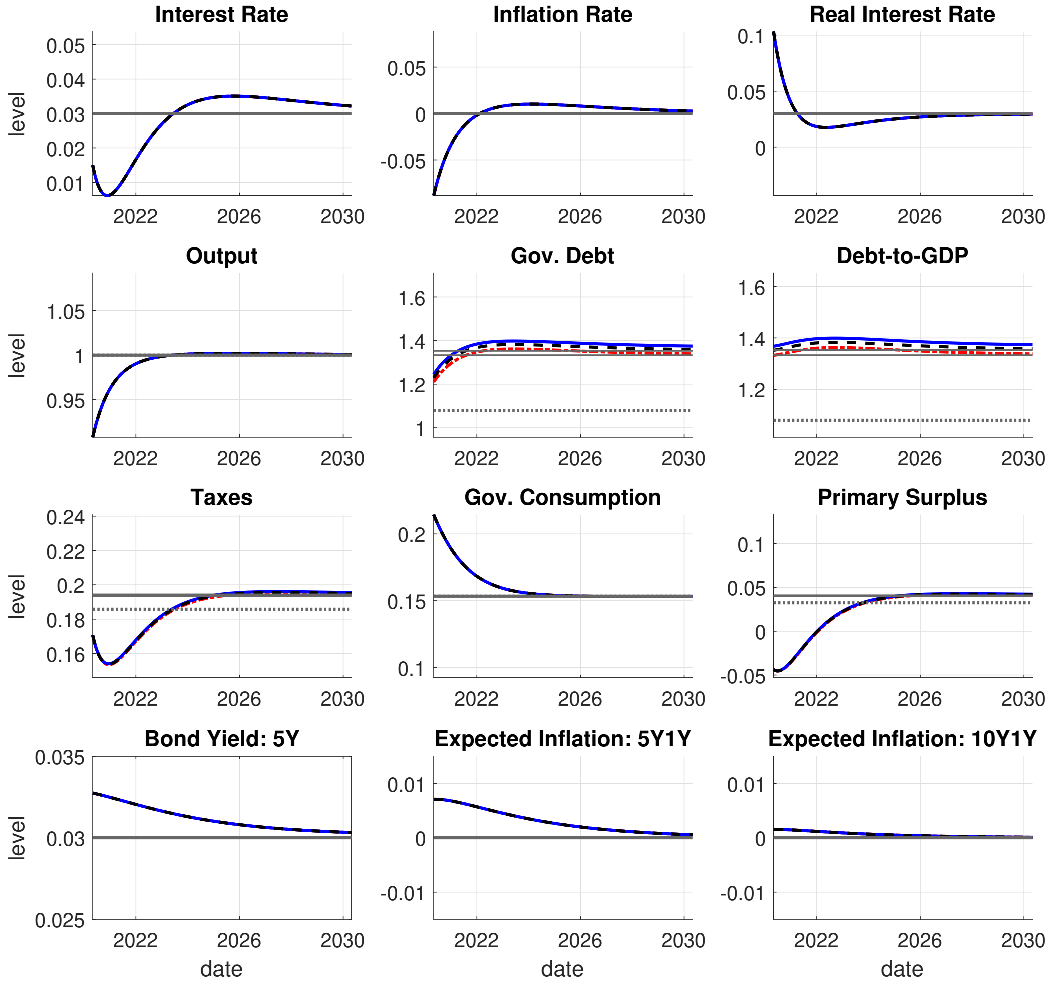

Figure D.21: Fully Funded CARES Act. Long-Term: $v_{ss}^{new} = 1.35$ for $\alpha = 2.11$. Average: $v_{ss}^{new} = 1.37$ for $\alpha = 2.23$. Short-Term: $v_{ss}^{new} = 1.33$ for $\alpha = 1.94$.

Table D.12: Inflation decomposition (17) for the CARES Act shock in Figure D.16.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru} i_u du$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	0	−1.46	−10.60	1.46	−10.60
Average	0	−1.46	−10.52	2.58	−11.64
Short-Term	0	−1.46	−10.72	0	−9.27

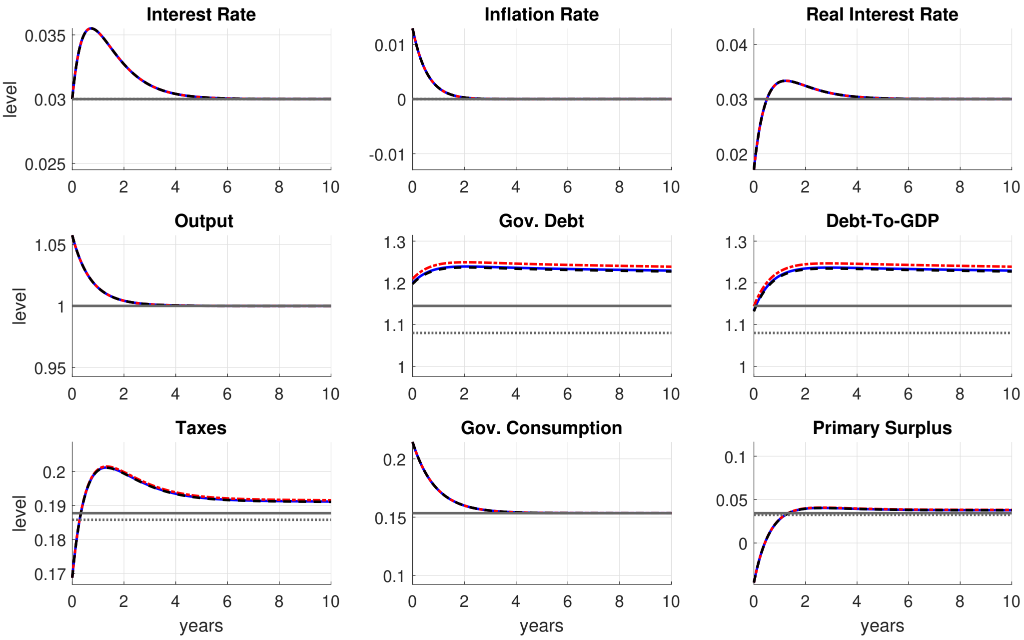

Figure D.22: CARES Act shock in the simple NK model with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$, $\varphi_y = -s_g$. To obtain the monetary regime, we further set $\tau_a = 0.04$ and $\phi_\pi = 1.6$ and. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

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